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Aerodynamic Preliminary Analysis System II Part I - Theory

E. Bonner, W Clever, and K Dunn

ROCKWELL INTERNATIONAL CORPORATION
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AERODYNAMIC PRELIMINARY ANALYSIS SYSTEM II

PART I THEORY

By E. Bonner, W. Clever, K. Dunn

North American Aircraft Operations, Rockwell International

SUMMARY

An aerodynamic analysis system based on potential theory at subsonic/supersonic speeds and impact type finite element solutions at hypersonic conditions is described. Three-dimensional configurations having multiple non-planar surfaces of arbitrary planform and bodies of non-circular contour may be analyzed. Static, rotary, and control longitudinal and lateral-directional characteristics may be generated.

The analysis has been implemented on a time sharing system in conjunction with an input tablet digitizer and an interactive graphics input/output display and editing terminal to maximize its responsiveness to the preliminary analysis problem. Computation times on an IBM 3081 are typically less than one minute of CPU/Mach number at subsonic, supersonic or hypersonic speeds. Computation times on PRIME 850 or a VAX 11/785 are about fifteen times longer than on the IBM. The program provides an efficient analysis for systematically performing various aerodynamic configuration tradeoff and evaluation studies.

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INTRODUCTION

Aerodynamic numerical analysis has developed to a point where evaluation of complete aircraft configurations by a single program is possible. Programs designed for this purpose in fact currently exist, but are limited in scope and abound with subtleties requiring the user to be highly experienced. Many of the difficulties are attributable to the numerical sensitivity of the associated solution. In preliminary design stages, some degree of approximation is acceptable in the interest of modest turn-around time, reduced computational costs, simplification of input, and stability and generality of results. The importance of short elapsed time stems from the necessity to systematically survey a large number of candidate advanced configurations or major component geometric parameters in a timely manner. Modest computational cost allows a greater number of configurations and/or conditions to be economically investigated.

One approach in this spirit is to employ panel approximations which reduce the number of simultaneous equations required to satisfy flow boundary conditions. Surface chord plane formulations, locally two dimensional crossflow body solutions and non-interfering panel simplifications are examples of approximations which can be used for this purpose. An alternative approach is to use surface chord plane formulations again for thin surfaces which can carry lift and surface panels for thick body type regions.

Finite element analysis when combined with realistic assessment of limitations and estimated viscous characteristics provides a valuable tool for analyzing general aircraft configurations and aerodynamic interactions at modest attitudes for subsonic/supersonic speeds and evaluation of compressible non-linearities at high Mach numbers.

LIST OF SYMBOLS

A	Projected oblique cross-section area
A_{ij}	Influence coefficient. Normalwash at control point i due to vortex panel j of unit strength
A_i	Area of quadrilateral panel i
b	Reference span
c	Local chord
\bar{c}	Reference chord
c_{AVG}	Average chord
C_d	Section drag coefficient
C_D	Drag coefficient
C_F	Flat plate average skin friction coefficient
C_i	Boundary condition for control point i
C_ℓ	Section lift coefficient
C_ℓ, C_m, C_n	Rolling, pitching and yawing moment coefficients
C_L	Lift coefficient
C_n	Section normal force coefficient
C_p	Pressure coefficient $(P - P_\infty)/q$
C_{pNET}	Net pressure coefficient $(P_\ell - P_u)/q$ and vortex panel strength
C_s	Leading edge suction coefficient
C_T	Leading edge thrust coefficient
C_x, C_y, C_z	Axial, side, normal force coefficient

C^*	$(\mu^* T_\infty)/(\mu_\infty T^*)$
F_x, F_y, F_z	Force components
$g(x)$	Axisymmetric outer solution to potential equation
h	Radius of curvature of cross-sectional boundary
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors in x, y, z direction respectively
K	Drag due to lift factor or skin friction thickness correction factor
K_s	Equivalent distributed sand grain height or attainable suction fraction
ℓ	Effective length
$\ell(i, n)$	Length of segment i, i+1 of contour C_n
L	Equivalent body length or geometric length
L/d	Body fineness ratio
M	Mach number
M_x, M_y, M_z	Moment components
\vec{n}	Unit normal
p, q, r	Rolling, pitching and yawing velocity about x, y and z
$\hat{p}, \hat{q}, \hat{r}$	Nondimensional angular velocities $pb/2U$, $q\bar{c}/2U$ and $rb/2U$
P	Static pressure
P_r	Prandtl number
q	Free stream dynamic pressure, $1/(2\rho U^2)$
r	Recovery factor
R	Unit Reynolds number or radius of curvature
$R_{[]}$	Reynolds number based on []
\bar{R}	Gas constant

s	Segment arc length
S	Body cross-sectional area or surface area
S_{REF}	Reference area
T	Static temperature T^0 , R , or tangent of quadrilateral panel leading edge sweep
t/c	Airfoil thickness ratio
u,v,w	x,y,z nondimensional perturbation velocity components
U,V	Freestream velocity
V_j	Jet velocity
W	Complex potential function
x,y,z	Body axis Cartesian coordinate system
x,r,θ	Body axis Cylindrical coordinate system
Z	Complex number $y+iz$
α	Angle of attack
α_i	Local angle of attack at surface control point i
β	Angle of sideslip or $[1-M^2]^{1/2}$
γ	Vorticity strength per unit length or ratio of specific heats
Γ	Horseshoe vortex strength in Trefftz plane
δ	Deflection or impact angle
η	Lateral surface coordinate
δ_{ij}	Kroneker delta: $\begin{matrix} 0 & i \neq j \\ 1 & i = j \end{matrix}$
δ_j	Jet deflection angle relative to trailing edge
δ_{jT}	Total jet deflection angle
$\delta\nu/\delta x$	Body slope
θ	Dihedral angle of quadrilateral panel or boundary layer momentum thickness
Λ	Sweep angle

μ	Absolute viscosity
ν	Kinematic viscosity, μ/ρ
ρ	Density
σ	Source density
T	Side edge rotation factor
ϕ	Perturbation velocity potential
Φ	Total velocity potential
ψ	See figure 4
Ω	Leading edge rotation factor

Subscript

c	camber
CG	center of gravity
e	edge conditions
F	friction
ℓ	lower surface
LE	leading edge
r	recovery
t	thickness
T	tip
TRAN	transition point
u	upper surface
v	vortex
w	wave
∞	freestream condition

Superscripts

'	first derivative or quantity based on effective origin
"	second derivative
*	Eckert reference temperature condition
\rightarrow	vector quantity

SUBSONIC/SUPERSONIC

The arbitrary configurations which may be treated by the analysis are simulated by a distribution of source and vortex singularities. Each of these singularities satisfies the linearized small perturbation potential equation of motion

$$\beta^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

The singularity strengths are obtained by satisfying the condition that the flow is tangent to the local surface:

$$\partial\Phi/\partial n = 0$$

All of the resulting velocities and pressures throughout the flow may be obtained when the singularity strengths are known. A configuration is composed of bodies, interference shells and aerodynamic surfaces (wings, canards, tails etc.). There are two alternative types of singularities used to represent the configuration. Figure 1 shows the first type, which can be used at all Mach numbers, and figure 2 shows an alternative method, which can be used only at subsonic Mach numbers.

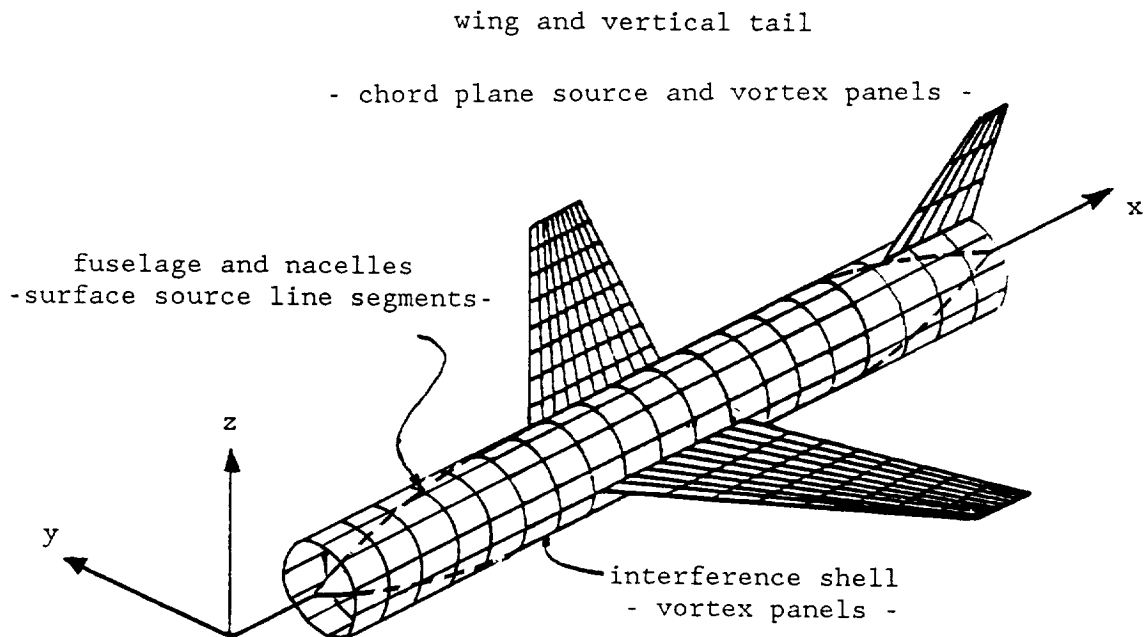


Figure 1 A. Singularities Used to Simulate a Configuration.

In the first method, the first step in the solution procedure consists of obtaining the strengths of the singularities simulating the fuselage and nacelles, from an isolated body solution. The present analysis uses slender-body theory to predict the surface and near field properties. The solution is composed of a compressible axisymmetric component for a body of revolution of the same cross-sectional area and an incompressible crossflow component, ϕ , satisfying the local three dimensional boundary conditions in the (y,z) plane. The crossflow is a solution of Laplace's equation

$$\varphi_{yy} + \varphi_{zz} = 0$$

A two-dimensional surface source distribution formulation is used to obtain this solution. When the body singularity strengths are determined, the perturbation velocities which they induce on the aerodynamic surfaces, or other regions of the field, are evaluated.

The assumptions of thin airfoil theory allow the effects of thickness and lift on aerodynamic surfaces to be considered independently. Therefore, the effects of the aerodynamic surfaces can be simulated by source and vortex singularities accounting for the effects of thickness and lift, respectively. The source and vortex distributions used in this program are in the form of quadrilateral panels having a constant source or vortex strength. The vortex panels have a system of trailing vortices extending undeflected to downstream infinity. The use of a chordwise linearly varying source panel is provided as an option to eliminate singularities associated with sonic panel edges at supersonic Mach numbers. The panels are planar, that is they have no incidence to the free stream (although dihedral may be included), since thin airfoil theory allows the transfer of the singularities and boundary conditions to the plane of the mean chord. These boundary conditions are satisfied at a single control point on each panel. For thickness, the control point is located at the panel centroid while the effects of twist, camber, and angle of attack are satisfied at the spanwise centroid of each vortex panel and at 87.5 percent of its chord.

A cylindrical, non-circular, interference shell, composed entirely of vortex panels, is used to account for the interference effects of the aerodynamic surfaces on the fuselage and nacelles. The boundary conditions on an interference shell are such that the velocity normal to the shell induced by all singularities, except those of the body which it surrounds, is zero. The boundary conditions are satisfied at the usual control points for vortex panels.

The second alternative method uses constant doublet panels and constant source panels to represent the body surface. These panels can be of an arbitrary quadrilateral shape and may be inclined to the direction of flow. The aerodynamic surfaces are represented by the same type of chord plane source and vortex panels as were used in the first method.

Alternative method for subsonic flow only

wing and vertical tail
- chord plane source and vortex panels -

fuselage
- surface source and doublet panels -

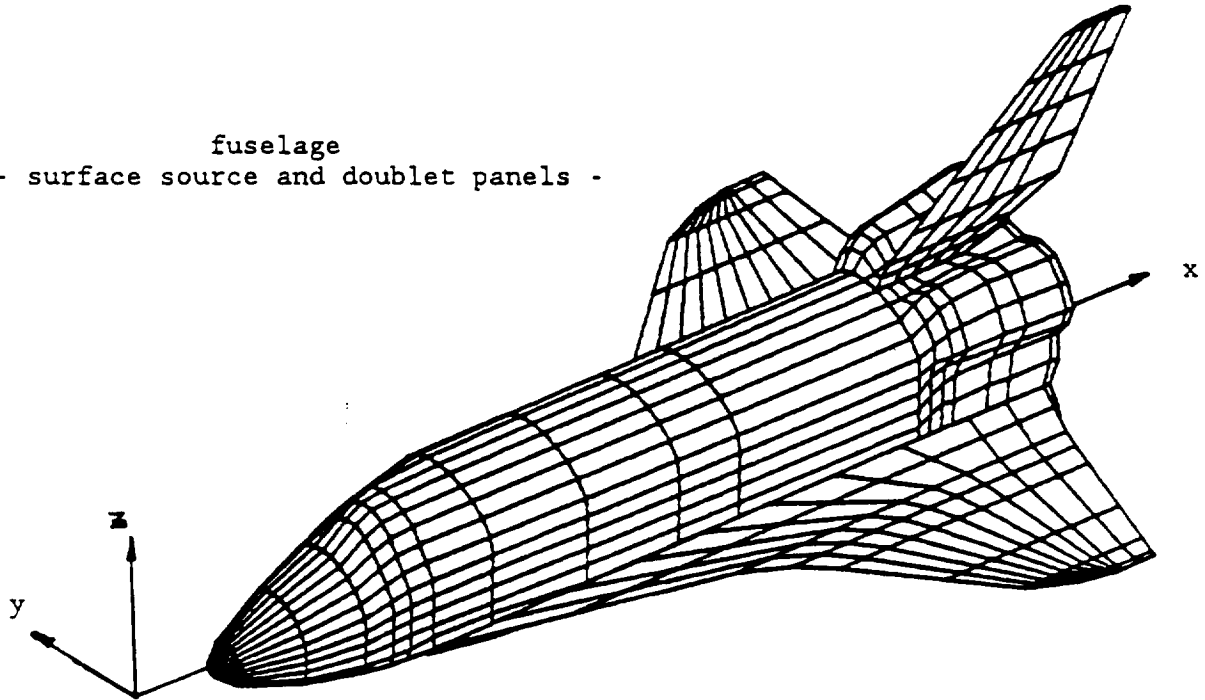


Figure 1 B. Singularities Used to Simulate a Configuration ($M_\infty < 1$).

This second method can be used at subsonic Mach numbers only. At supersonic Mach numbers, the doublet panels, which are equivalent to quadrilateral vortices, produce infinite perturbation velocities in certain regions of the flow, and thus cannot be used. The body source and doublet strengths are chosen to satisfy both an arbitrary normal velocity boundary condition on the body,

$$\partial\Phi/\partial n = V_n$$

and to have zero perturbation potential in the entire region interior to the body surface.

$$\Phi = 0$$

The following sections define the details of the solution procedure. Included are discussions of the isolated body analysis, surface finite element analysis considering edge effects, and evaluation of aerodynamic characteristics including drag. References are cited for the reader interested in further pursuing a particular point.

SLENDER BODY SOLUTION

According to slender body theory^{1,2} the flow disturbance near a sufficiently regular three-dimensional body may be represented by a perturbation potential of the form

$$\phi = \varphi(y, z; x) + g(x) \quad (1)$$

$\varphi(y, z; x)$ is a solution of the 2-D Laplace equation in the (y, z) cross flow plane satisfying the following boundary conditions

$$\nabla\varphi = \vec{j}v + \vec{k}w = 0$$

$$\partial\phi/\partial n = 0, \quad \text{on } C(x) \quad (2)$$

$C(x)$ and n are defined in figure 2. A general solution for ϕ may be written as the real part of a complex potential function $W(Z)$ with $Z = y + iz$.

$$\varphi = R_e W = R_e \left[A_0(x) \ln Z + \sum_{n=1}^{\infty} A_n(x) Z^{-n} \right]$$

A useful alternative representation of ϕ and W is obtainable with the aid of Green's theorem.³

$$\varphi = R_e W = -2 R_e \oint_{C(x)} \sigma(\zeta) \ln(Z - \zeta) ds \quad (3)$$

where $\sigma(\zeta)$ is a "source" density for values of $\zeta = y_c + iz_c$, (y_c, z_c) being coordinates of a point on the contour $C(x)$.

The function $g(x)$ obtained by matching ϕ of equation (1) which is valid in the neighborhood of the body with an appropriate "outer" solution. $g(x)$ is then found to depend explicitly on the Mach number M and longitudinal variation of cross-sectional areas $S(x)$

$$g(x) = 1/(2\pi) [S'(x) \ln(0.5\beta) - 1/2 \int_0^x S''(t) \ln(x-t) dt + 1/2 \int_x^1 S''(t) \ln(t-x) dt - 1/2 S'(0) \ln x - 1/2 S'(1) \ln(1-x)] \quad M < 1 \quad (4)$$

$$g(x) = 1/(2\pi) [S'(x) \ln(0.5\beta) - \int_0^x S''(t) \ln(x-t) dt] \quad M > 1$$

The body axis perturbation velocities are obtained by differentiation of equation (1)

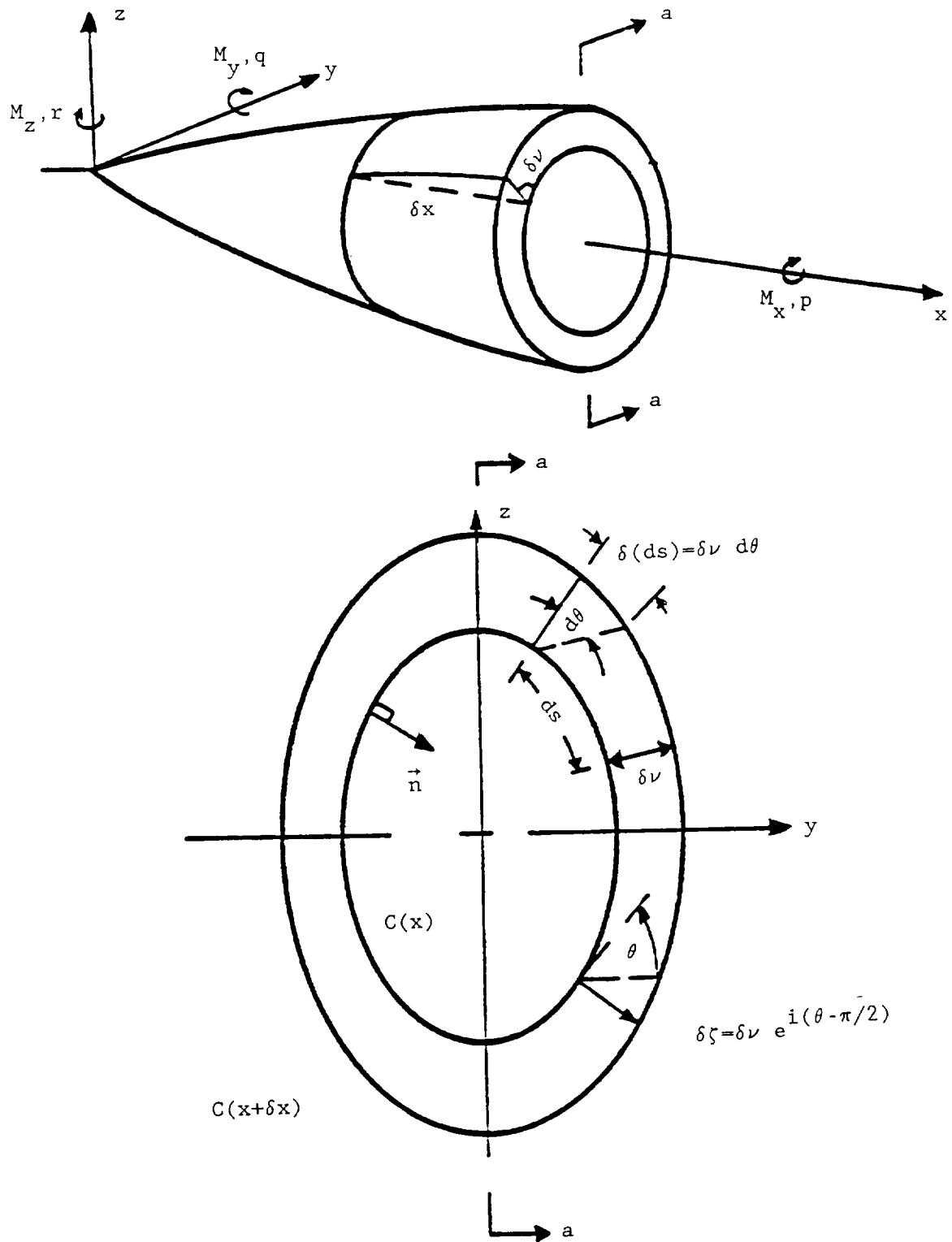


Figure 2. Body Slope and Cross-sectional Variables.

$$u = \phi_x = \varphi_x + g'(x)$$

$$v = \phi_y$$

$$w = \phi_z$$

At supersonic speeds, zone of influence considerations require that $u = v = w = 0$ for $x - \beta r < 0$.

Solution of the preceding equations is based on an extension of the method of reference 3.

CROSS FLOW COMPONENT

The reduction of computations to a numerical procedure utilizes the integral representation of φ given in equation (3) by discretization of the cross-sectional boundary into a large number of short linear segments (figure 3) over each of which the source density σ is assumed constant at a value determined by boundary conditions.

Computation of $\sigma(i,n)$ over the segment $i, i+1$ proceeds by applying the boundary condition equation (2) at each segment of C_n . If $\nabla\varphi = \vec{q} = \vec{j}v + \vec{k}w$ represents the velocity vector, the corresponding complex velocity in the cross flow plane is obtained by differentiation of W in equation (3) with respect to Z :

$$v - iw = -2\oint \sigma(\zeta)/(Z-\zeta) ds \quad (5)$$

The contribution by the sources located on segment $i, i+1$ to the velocity at $\tilde{P}_{j,n}$ is first evaluated. Noting that $i, i+1$ makes an angle $\theta(i,n)$ with respect to the horizontal axis, we have

$$d\zeta = ds e^{i\theta(i,n)}$$

and the contribution of the integral in equation (5) may be written:

$$\Delta[v(j,n) - iw(j,n)] = -2\sigma(i,n)e^{-i\theta(i,n)} \int_{\zeta_{i,n}}^{\zeta_{i+1,n}} [Z_{j,n} - \zeta]^{-1} d\zeta$$

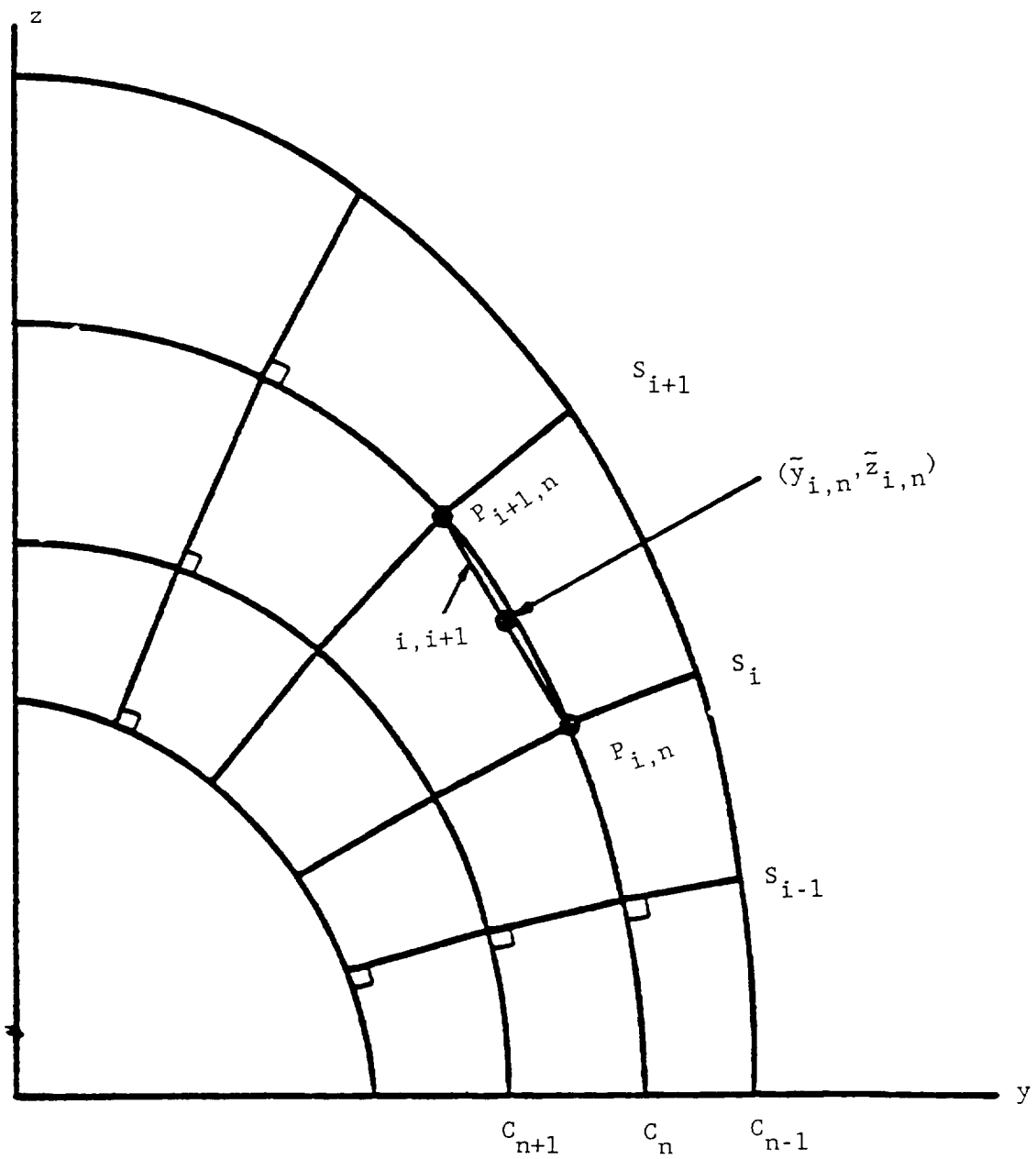


Figure 3. Cross-section Boundary Segmenting Scheme.

After integration of the last term and summation over all contributing segments, the result may be written

$$v(j,n) - iw(j,n) = -2 \sum_i \sigma(i,n) e^{-i\theta(i,n)} \left\{ \ln[R(i+1,j,n)/R(i,j,n)] + i\delta(i,j,n) \right\} \quad (6)$$

in which referring to figure 4, the quantities $R(i,j,n)$ and $\delta(i,j,n)$ are defined by the relationships

$$R(i,j,n) e^{i\psi(i,j,n)} = \tilde{z}_{j,n} - \zeta_{i,n}$$

$$\delta(i,j,n) = \tilde{\psi}(i,j,n) - \psi(i,j,n)$$

To insure uniqueness of the complex velocity, care must be exercised in assigning values to the angles $\psi(i,j,n)$ and $\tilde{\psi}(i,j,n)$. Referring to figure 4, these are measured counter-clockwise from the positive y-axis so that when facing $P_{i,n}$ to $P_{i+1,n}$, a point $\tilde{P}_{j,n}$ just to the left of $i,i+1$ shall define an angle $\psi(i,j,n) = \theta(i,n)$. As $\tilde{P}_{j,n}$ traverses a path around $P_{i,n}$ to a point just to the right of $i,i+1$, $\psi(i,j,n)$ increases from $\theta(i,n)$ to $\theta(i,n) + 2\pi$. The same holds true for $\tilde{\psi}(i,j,n)$ as $\tilde{P}_{j,n}$ traverses a path around $P_{i+1,n}$. In consequence of these definitions $\delta(i,j,n)$ becomes $-\pi$ when approaching $i,i+1$ from the right and π when approaching from the left. This discontinuity reflects that exhibited by the stream function upon traversing any closed path which encloses a distribution of finite sources.

From the boundary condition equation (2), we have

$$-(\partial\varphi/\partial n)_{j,n} = v(j,n)\sin\theta(j,n) - w(j,n)\cos\theta(j,n)$$

After substitution of v and w from equation (6), this last expression becomes

$$-(\partial\varphi/\partial n)_{j,n} = \sum_i a(j,i)\sigma(i,n) \quad (7)$$

where

$$a(j,i) = 2 \left\{ \sin[\theta(j,n) - \theta(i,n)] \ln[R(i+1,j,n)/R(i,j,n)] + \delta(i,j,n) \cos[\theta(j,n) - \theta(i,n)] \right\}$$

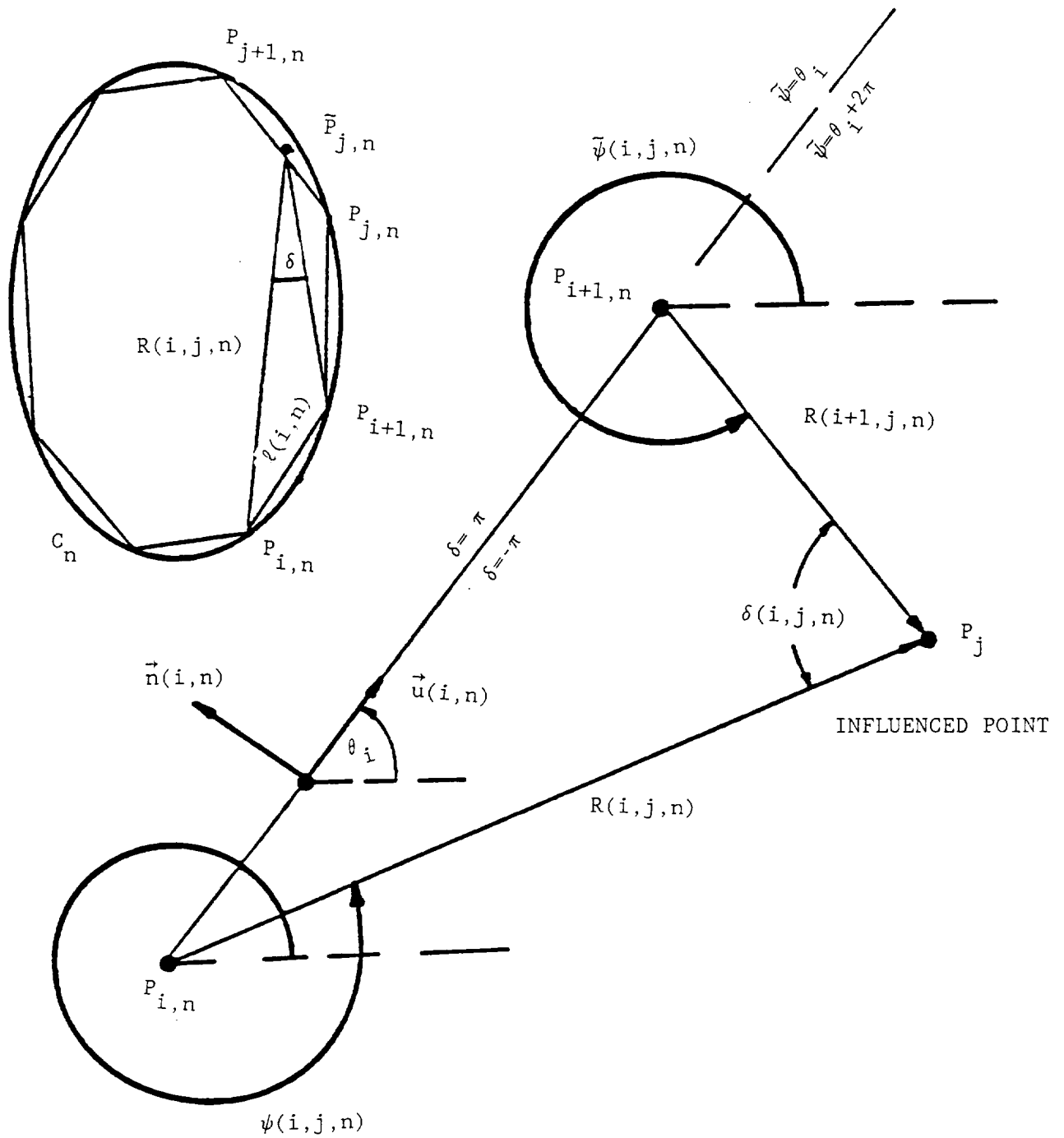


Figure 4. Details of Variables Pertaining to Segment $i, i+1$ of Boundary C_n .

The surface normal perturbation velocity - $(\partial\varphi/\partial n)_{j,n}$ may be written in terms of the body slope $(\partial\nu/\partial x)_{j,n}$, the angles of attack α , and sideslip β and the angular velocities p, q, r as

$$- (\partial\varphi/\partial n)_{j,n} = (\partial\nu/\partial x)_{j,n} + [\alpha + q(x-x_{cg})/U + py/U] \cos\theta(j,n) \\ + [\beta - r(x-x_{cg})/U + p(z-z_{cg})/U] \sin\theta(j,n)$$

Satisfying equation 7 at each of the points $\tilde{P}_{j,n}$ on a given contour boundary yields a set of equations for $\sigma(i,n)$.

AXISYMMETRIC COMPONENT

Differentiation of $g(x)$ must be carried out with due concern for the nature of the improper integrals appearing in equation (4). The result is

$$g'(x_n) = 1/(4\pi) \left\{ S''(x_n) \ln[0.25(1-M^2)] + I_n(x_n) - J_n(x_n) \right. \\ \left. - S'(0)/x_n + S'(1)/(1-x_n) - S''(0) \ln x_n - S''(1) \ln(1-x_n) \right\} \quad M < 1$$

$$g'(x_n) = 1/(2\pi) \left\{ 1/2 S''(x_n) \ln[0.25(M^2-1)] - J_n(x_n) - S''(0) \ln x_n \right\} \quad M > 1$$

where

$$I_n(x_n) = \int_{x_n}^1 \ln(x_n - t) S''''(t) dt = \sum_{m=n}^{n-1} [S''_{m+1} - S''_m] \ln(\tilde{x}_m - x_n)$$

$$J_n(x_n) = \int_0^{x_n} \ln(x_n - t) S''''(t) dt = \sum_{m=0}^{n-1} [S''_{m+1} - S''_m] \ln(x_n - \tilde{x}_m)$$

$$\tilde{x}_m = (x_{m+1} + x_m)/2$$

To compute the second derivatives of the equivalent body cross-sectional area required for $g'(x)$, the first derivatives at \tilde{x}_m are found by finite differences between x_m and x_{m+1} . Second derivatives $S''(\tilde{x}_m)$ at $\tilde{x}_m = (\tilde{x}_{m+1} + \tilde{x}_m)/2$ are then found by finite differences between S' at \tilde{x}_m and \tilde{x}_{m+1} . Finally $S''(x_m)$ is determined by linear interpolation of $S''(\tilde{x}_m)$ between \tilde{x}_m and \tilde{x}_{m+1} .

PERTURBATION VELOCITIES

The axial velocity u depends on $\partial\phi/\partial x$ and the axisymmetric solution $g'(x)$. $\partial\phi/\partial x$ is obtained by differentiation of the integral in equation (3) to first obtain an exact expression which is then approximated by evaluating the result over the segmented boundary.

The derivation of $\partial\phi/\partial x$ must take into account the fact that the path of integration in equation (3) is a function of x . Referring to figure 2 increments of a dependent variable taken along $C(x)$ are denoted by $d(\)$ and increments taken normal to C are denoted by $\delta(\)$. Differentiation of equation (3) then yields

$$\begin{aligned} \partial\phi/\partial x = -2 R_e [& \oint (\delta\sigma/\delta x) \ln(Z-\zeta) ds - \oint \sigma(\zeta)/(Z-\zeta) (\delta\zeta/\delta x) ds \\ & + \oint \sigma(\zeta) \ln(Z-\zeta) (\delta(ds)/\delta x)] \end{aligned} \quad (8)$$

From figure 2

$$\delta(ds) = \delta\nu d\theta = \delta\nu ds/h(\zeta) \quad (9)$$

where $h(\zeta)$ is the radius of curvature of $C(x)$ at ζ . In addition, we have from figure 2

$$\delta\zeta/\delta x = \delta\nu/\delta x e^{i(\theta - 0.5\pi)} \quad (10)$$

To evaluate $\delta\sigma/\delta x$ we note,

$$\delta\sigma/\delta x = \lim_{\delta x \rightarrow 0} [\sigma(i, n+1) - \sigma(i, n)]/\delta x \quad (11)$$

Introducing equations (9), (10), and (11) into equation (8),

$$\partial\phi/\partial x = -2 R_e \left\{ \oint [(\delta\sigma/\delta x)_0 + \sigma/h \delta\nu/\delta x] \ln(Z-\zeta) ds + i \oint [\sigma(\delta\nu/\delta x)] d\zeta/(Z-\zeta) \right\}$$

Again, assuming that quantities in the brackets of the integrands are constant over $i, i+1$,

$$\begin{aligned} (\partial\phi/\partial x)_{j,n} = 2 \sum_i \left\{ [(\delta\sigma/\delta x)_0 + \sigma/h(\delta\nu/\delta x)]_{i,n} \Delta\phi(i,j,n)/\sigma(i,n) \right. \\ \left. - \sigma(i,n) (\delta\nu/\delta x)_{i,n} \delta(i,j,n) \right\} \end{aligned}$$

where

$$\begin{aligned} \Delta\phi(i,j,n)/\sigma(i,n) = \left\{ \vec{R}(i+1,j,n) \cdot \vec{u}(i,n) \ln R(i+1,j,n) \right. \\ - \vec{R}(i,j,n) \cdot \vec{u}(i,n) \ln R(i,j,n) \\ \left. - \vec{R}(i,j,n) \cdot \vec{n}(i,n) \delta(i,j,n) + \ell(i,n) \right\} \end{aligned}$$

The radius of curvature $h(i,n)$ and the derivatives $\delta\sigma/\delta x$, $\delta\nu/\delta x$ are approximated at the mid-points of the segments $i, i+1$ as follows

$\delta\sigma/\delta x$ - the derivative at the mid-point \tilde{x}_n of the interval x_n, x_{n+1} is set equal to the divided difference between $\sigma(i,n)$ and $\sigma(i,n+1)$. Linear interpolation between these derivatives then yields $\delta\sigma/\delta x$ at x_n .

$\delta\nu/\delta x$ - referring to figure 5, the displacement $\delta\eta$ is determined by linear interpolation between $\delta\zeta_{i,n}$ and $\delta\zeta_{i+1,n}$.

$\delta\eta/(x_{n+1}-x_n)$ then represents $\delta\nu/\delta x$ at \tilde{x}_n . Linear interpolation between the stations x'_n then yields $\delta\nu/\delta x$ at x_n .

$1/h$ - θ at $P_{i,n}$ is determined by interpolation between values of $\theta(i,n)$ at $\tilde{P}_{i,n}$. The curvature $1/h$ at $\tilde{P}_{i,n}$ is then set equal to the divided difference between θ at $P_{i+1,n}$ and θ at $P_{i,n}$.

The lateral and vertical perturbation velocities, v and w , are obtained from

$$v - iw = -2 \oint \sigma(\zeta)/(Z-\zeta) ds$$

Integration over the boundary with constant segment source density yields:

$$v(j,n) - iw(j,n) = 2 \sum_i \sigma(i,n) e^{i\theta(i,n)} \left\{ \ln[R(i+1,j,n)/R(i,j,n)] - i\delta(i,j,n) \right\}$$

Thus

$$v = \phi_y = 2 \sum_i \sigma(i,n) \left\{ \ln[R(i+1,j,n)/R(i,j,n)] \cos \theta(i,n) - \delta(i,j,n) \sin \theta(i,n) \right\}$$

$$w = \phi_z = 2 \sum_i \sigma(i,n) \left\{ \ln[R(i+1,j,n)/R(i,j,n)] \sin \theta(i,n) - \delta(i,j,n) \cos \theta(i,n) \right\}$$

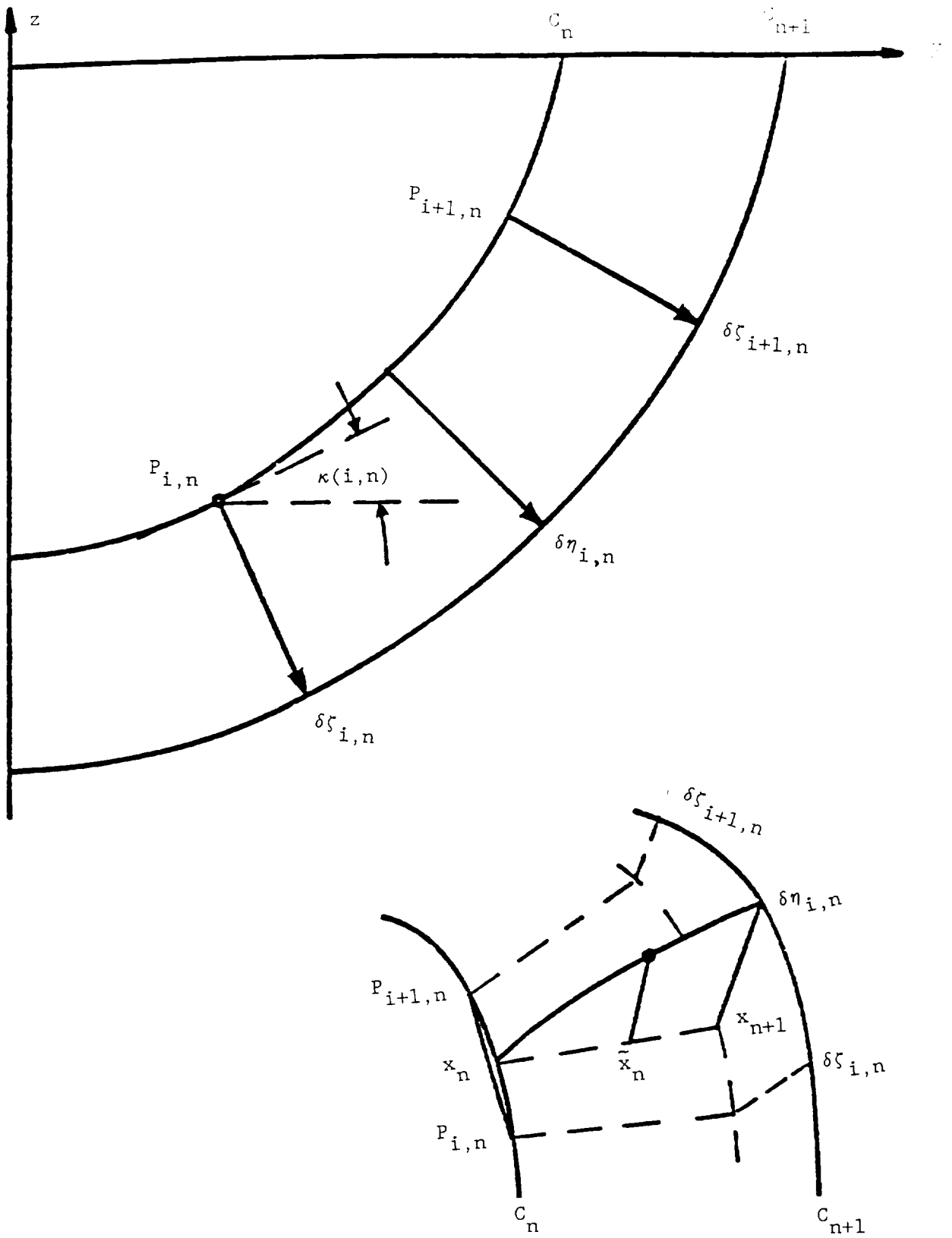


Figure 5. Interpolation Procedure for Determination of $(\delta\nu/\delta x)_{i,n}$.

SURFACE SOLUTION

CHORD PLANE SOURCE AND VORTEX PANELS

The wing, canard, vertical and horizontal tail are simulated by a system of swept tapered chord plane source and vortex panels with two edges parallel to the free stream. The coordinates of the panel corners are specified with respect to an (x,y,z) system having its x -axis in the free stream direction and its z -axis in the lift direction. The panel influence equations are written in terms of a coordinate system having a z -axis normal to the panel and an x -axis along one of the two parallel edges. A coordinate transformation is necessary to obtain the coordinates in the panel reference system. If the plane of the panel is inclined at an angle θ_p with respect to the y,z plane, a transformation into the panel coordinate system (x_p, y_p, z_p) is accomplished as shown in figure 6.

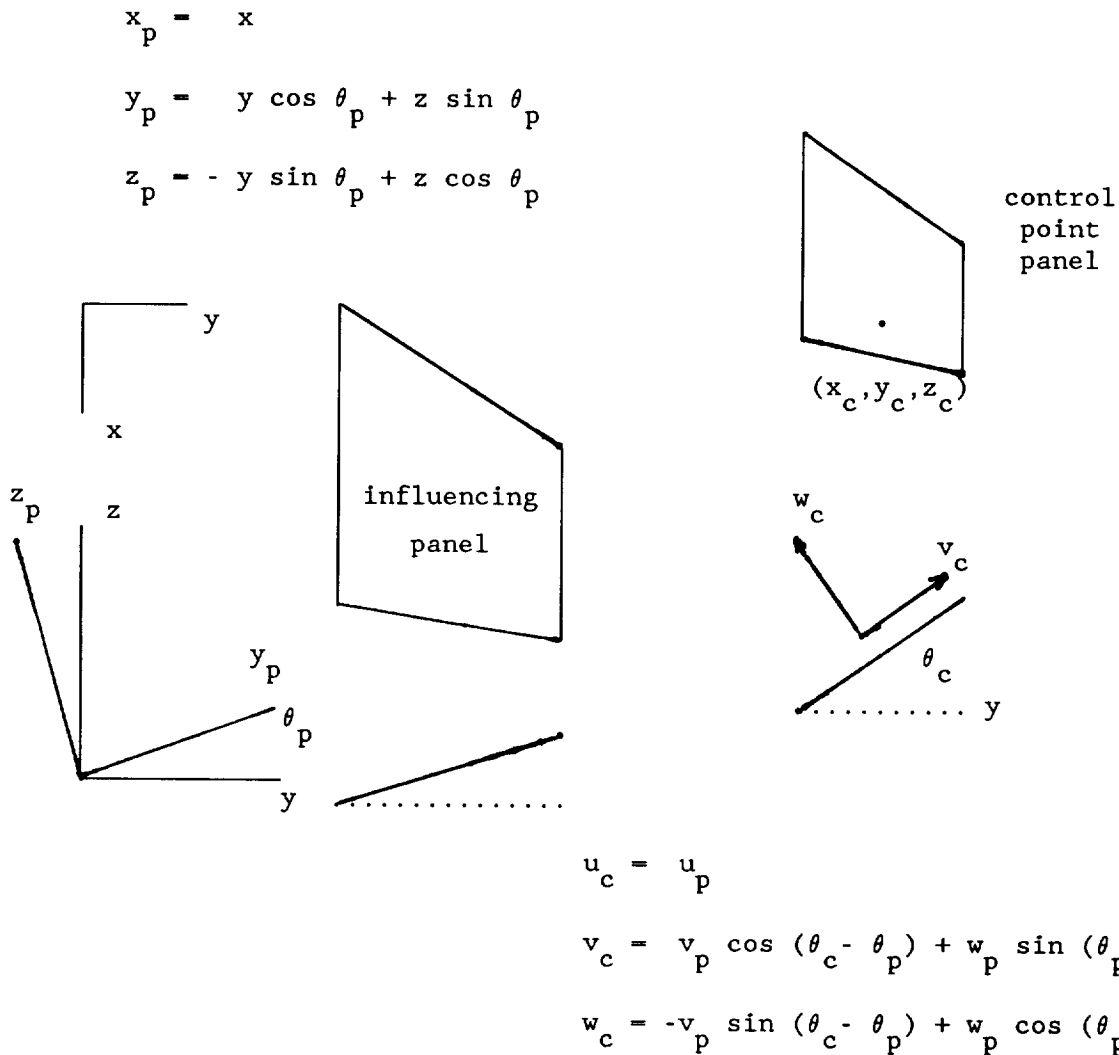


Figure 6. Coordinate Transformation in Panel Reference System.

A transformation of the (u_p, v_p, w_p) velocities into the coordinate system of the panel on which the control point is located (u_c, v_c, w_c) results in the axial, binormal and normal velocities induced on the panel.

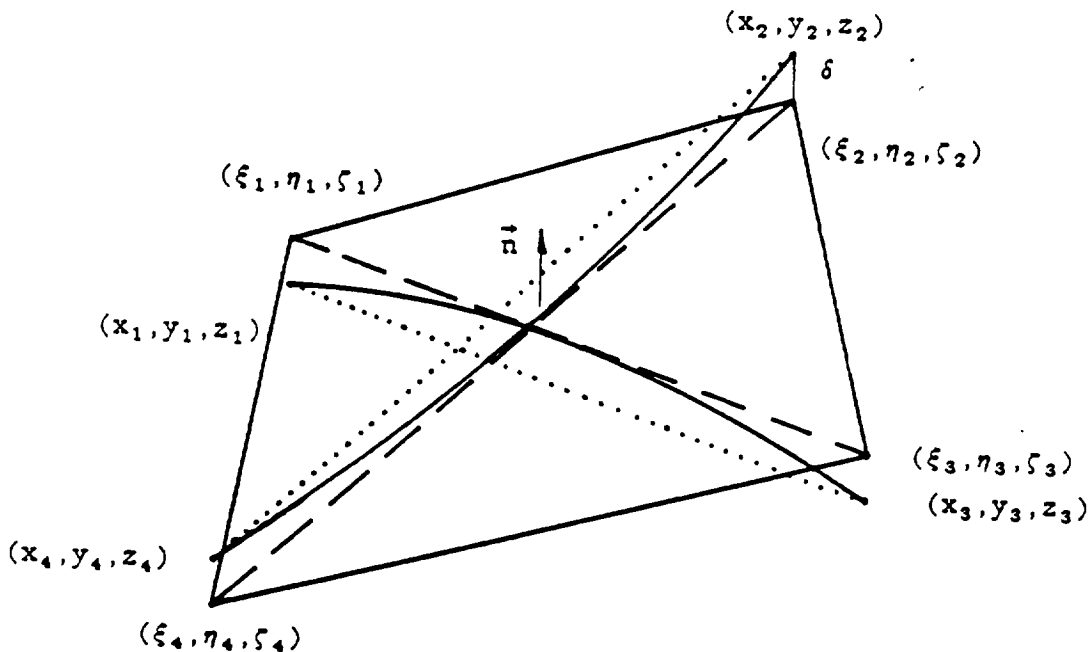
For the image of the influencing panel, the signs of y_c , θ_c and v_c are changed while using the same calculation procedure.

QUADRILATERAL SOURCE AND DOUBLET PANELS (PANELED BODIES)

For subsonic Mach numbers the body may be represented by a system of planar quadrilateral constant source and constant doublet panels. Since four points arbitrarily selected on a surface may not lie in the same plane, a mean surface through the four points is selected to represent the panel.

Let (x_i, y_i, z_i) represent the four points on the body surface,

and (ξ_i, η_i, ζ_i) represent the four points on the mean surface.



This mean surface is chosen in the following manner.

1. The direction of the panel normal is found by taking the cross product of the vectors representing the diagonals.

$$\vec{n} = \frac{\vec{d}_{31} \times \vec{d}_{42}}{|\vec{d}_{31}| |\vec{d}_{42}|}$$

$$\vec{d}_{31} = (x_3 - x_1, y_3 - y_1, z_3 - z_1)$$

$$\vec{d}_{42} = (x_4 - x_2, y_4 - y_2, z_4 - z_2)$$

2. The out of plane distance, δ , is calculated using vectors determined by pairs of points.

$$\delta = \frac{1}{4} (\vec{s}_{12} + \vec{s}_{34}) \cdot \vec{n}$$

$$\vec{s}_{12} = (x_1 - x_2 , y_1 - y_2 , z_1 - z_2)$$

$$\vec{s}_{34} = (x_3 - x_4 , y_3 - y_4 , z_3 - z_4)$$

3. The coordinates of the mean surface are calculated by adding or subtracting $\delta \vec{n}$ from each of the corner points. i.e.

$$\begin{aligned} (\xi_1, \eta_1, \zeta_1) &= (x_1, y_1, z_1) - \delta (n_1, n_2, n_3) \\ (\xi_2, \eta_2, \zeta_2) &= (x_2, y_2, z_2) + \delta (n_1, n_2, n_3) \\ (\xi_3, \eta_3, \zeta_3) &= (x_3, y_3, z_3) - \delta (n_1, n_2, n_3) \\ (\xi_4, \eta_4, \zeta_4) &= (x_4, y_4, z_4) + \delta (n_1, n_2, n_3) \end{aligned}$$

The normal computed for these four points is the same as the normal for the original body points, since the diagonal vectors are the same. If a vector determined by the line segment joining any two of the four points is normal to \vec{n} , then the four points must lie in the same plane. This is easily shown to be true. From the above definitions,

$$0 = (\vec{\sigma}_{12} + \vec{\sigma}_{34}) \cdot \vec{n}$$

$$\vec{\sigma}_{12} = (\xi_1 - \xi_2 , \eta_1 - \eta_2 , \zeta_1 - \zeta_2)$$

$$\vec{\sigma}_{34} = (\xi_3 - \xi_4 , \eta_3 - \eta_4 , \zeta_3 - \zeta_4)$$

but

$$\begin{aligned} \vec{\sigma}_{34} &= (\xi_3 - \xi_4 , \eta_3 - \eta_4 , \zeta_3 - \zeta_4) \\ &= [(\xi_3 - \xi_1) - (\xi_4 - \xi_2) + (\xi_1 - \xi_2) \\ &\quad , (\eta_3 - \eta_1) - (\eta_4 - \eta_2) + (\eta_1 - \eta_2) \\ &\quad , (\zeta_3 - \zeta_1) - (\zeta_4 - \zeta_2) + (\zeta_1 - \zeta_2)] = \vec{d}_{31} - \vec{d}_{42} + \vec{\sigma}_{12} \end{aligned}$$

or

$$(\vec{\sigma}_{12} + \vec{\sigma}_{34}) \cdot \vec{n} = 2 \vec{\sigma}_{12} \cdot \vec{n} = 0$$

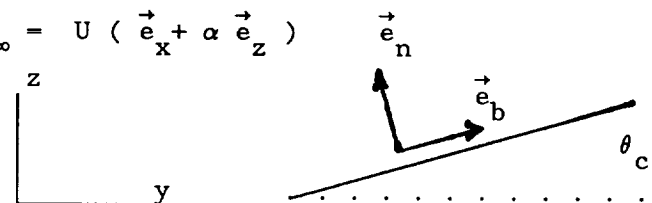
$$\text{similarly} \quad \vec{\sigma}_{12} \cdot \vec{n} = \vec{\sigma}_{23} \cdot \vec{n} = \vec{\sigma}_{34} \cdot \vec{n} = \vec{\sigma}_{41} \cdot \vec{n} = 0$$

and all four points must lie in the same plane.

BOUNDARY CONDITIONS

Vortex Panels

At the vortex panel control points the resultant velocity along the normal at a panel control point must be zero. Using a local coordinate system, with perturbation velocities (u_c, v_c, w_c) at the control points,

$$\vec{U}_\infty = U (\vec{e}_x + \alpha \vec{e}_z)$$


$$\begin{aligned} \vec{e}_z &= \vec{e}_n \cos \theta_c + \vec{e}_b \sin \theta_c \\ \vec{e}_y &= -\vec{e}_n \sin \theta_c + \vec{e}_b \cos \theta_c \end{aligned}$$

$$\begin{aligned} \vec{U} \cdot \vec{n} &= U [\vec{e}_x + u_c \vec{e}_x + (v_c + \alpha \sin \theta_c) \vec{e}_b + (w_c + \alpha \cos \theta_c) \vec{e}_n] \cdot \vec{n} \\ &= U [(1+u_c) \vec{e}_x \cdot \vec{n} + (w_c + \alpha \cos \theta_c)] = 0 \end{aligned}$$

$$\text{with } \vec{n} = [\vec{e}_n - (dz_c/dx)_i \vec{e}_x]$$

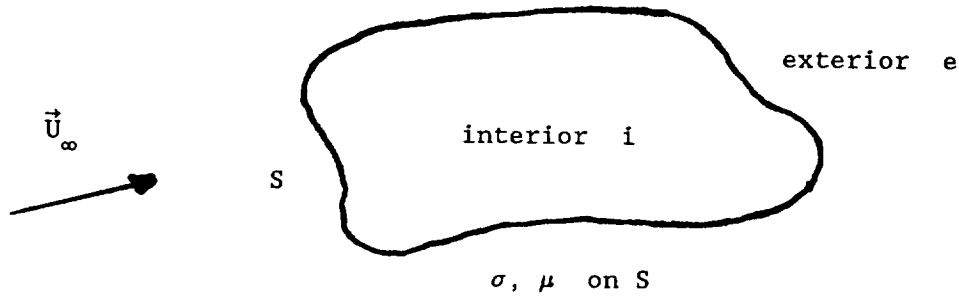
$$\text{For small perturbations } (1+u_c) \vec{e}_x \cdot \vec{n} \approx - (dz_c/dx)_i \quad \text{and} \quad \vec{e}_n \cdot \vec{n} \approx 1$$

$$\text{Therefore} \quad w_{c_i} = (dz_c/dx)_i - \alpha \cos \theta_c$$

Body Panels

The boundary condition on body panels will involve the normal component of velocity. If we set the normal component equal to zero, we have the usual flow tangency boundary condition. Nonzero normal components can be used for jets or inlets. Given the boundary condition on the surface and the field at infinity, the solution for the external flow is unique. It can be satisfied by an infinite number of combinations of source and doublet distributions on the surface. However each combination will result in a different field inside the body surface. Specifying an internal boundary condition will make the source and doublet distribution unique, and can have a powerful effect on the numerical behavior of a solution involving a finite number of elements. The internal boundary condition which we have chosen, with these numerical considerations in mind, has zero perturbation potential on the internal boundary, and therefore due to the nature of the governing equation, zero perturbation potential inside. Below, we will show that by first correctly choosing the surface source distribution, we can also satisfy the external normal velocity boundary condition by satisfying the internal surface boundary condition on ϕ .

Consider a closed region determined by the surface S . Let the surface have a distribution of sources and doublets with local strength σ and μ . The surface, S , will divide the interior and exterior regions.



Define

- e = the subscript denoting properties exterior to S
- i = the subscript denoting properties interior to S
- \vec{n}_e = the external normal to S = (n_x, n_y, n_z)
- \vec{u} = $\nabla \phi$ = perturbation velocities due to σ and μ .

$\vec{U} \cdot \vec{n}_e = U_{n_e}$ = the prescribed normal velocity on S (exterior)

We can set the value of the surface source strengths to any value and still satisfy the external boundary condition.

We will set, $\hat{\sigma} = -\vec{U}_\infty \cdot \vec{n}_e + U_{n_e} = \frac{\sigma}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}}$

and adjust the value of μ , and any other singularity strengths, such that everywhere on the interior surface of S,

$$\phi_i = 0$$

Then in the entire region interior to S,

$$\phi_i = 0$$

and $\vec{u}_i = \nabla \phi_i = 0$

$$\vec{U}_i = \nabla \phi_i + \vec{U}_\infty = \vec{U}_\infty$$

Since the μ gives a continuous normal velocity across S, using Appendix C,

$$\vec{u}_e \cdot \vec{n}_e + \vec{u}_i \cdot \vec{n}_i = \vec{u}_e \cdot \vec{n}_e = \hat{\sigma} = -\vec{U}_\infty \cdot \vec{n}_e + U_{n_e}$$

or

$$(\vec{U}_\infty + \vec{u}_e) \cdot \vec{n}_e = U_{n_e} \quad \text{as required on S.}$$

Therefore the normal velocity boundary condition can be satisfied by substituting a boundary condition for ϕ on the internal boundary surface.

PANEL INFLUENCE COEFFICIENTS

Each of the panel types induces a perturbation potential everywhere in space. If panel j has unit strength, we can say it will induce the following velocities and velocity potential at the control point of panel i .

$$\begin{aligned}
 (A_{ij}^u, A_{ij}^v, A_{ij}^w, A_{ij}^\phi) & \quad \text{for vortex panels} \\
 (D_{ij}^u, D_{ij}^v, D_{ij}^w, D_{ij}^\phi) & \quad \text{for body doublet panels} \\
 (S_{ij}^u, S_{ij}^v, S_{ij}^w, S_{ij}^\phi) & \quad \text{for body source panels} \\
 (T_{ij}^u, T_{ij}^v, T_{ij}^w, T_{ij}^\phi) & \quad \text{for thickness source panels}
 \end{aligned}$$

Therefore, assuming there are ntv vortex panels and ntb body panels, and C_{pj} , μ_j , σ_j , τ_j , are the panel singularity strengths, the following set of panel influence coefficients can be written:

$$\begin{aligned}
 u_i &= \sum_{j=1}^{ntv} A_{ij}^u C_{pj} + \sum_{j=1}^{ntb} D_{ij}^u \mu_j + \sum_{j=1}^{ntb} S_{ij}^u \sigma_j + \sum_{j=1}^{ntv} T_{ij}^u \tau_j + u_{0_i} \\
 v_i &= \sum_{j=1}^{ntv} A_{ij}^v C_{pj} + \sum_{j=1}^{ntb} D_{ij}^v \mu_j + \sum_{j=1}^{ntb} S_{ij}^v \sigma_j + \sum_{j=1}^{ntv} T_{ij}^v \tau_j + v_{0_i} \\
 w_i &= \sum_{j=1}^{ntv} A_{ij}^w C_{pj} + \sum_{j=1}^{ntb} D_{ij}^w \mu_j + \sum_{j=1}^{ntb} S_{ij}^w \sigma_j + \sum_{j=1}^{ntv} T_{ij}^w \tau_j + w_{0_i} \\
 \phi_i &= \sum_{j=1}^{ntv} A_{ij}^\phi C_{pj} + \sum_{j=1}^{ntb} D_{ij}^\phi \mu_j + \sum_{j=1}^{ntb} S_{ij}^\phi \sigma_j + \sum_{j=1}^{ntv} T_{ij}^\phi \tau_j + \phi_{0_i}
 \end{aligned}$$

where $(u_{0_i}, v_{0_i}, w_{0_i}, \phi_{0_i})$ refer to the perturbations induced by any other body or source singularities, e.g. slender bodies.

PANEL SINGULARITY STRENGTHS

Source (Thickness) Panels

The source singularity strengths for thickness panels may be found directly by equating each source panel strength to the slope of the thickness distribution at its control point. For panel i

$$\tau_i = (dZ_t/dx)_i$$

where Z_t refers to the shape of the thickness distribution.

Body Source Panels

The source singularity strengths for body source panels are set to give the correct normal velocity boundary condition when the internal perturbation potential is zero. For panel i we set

$$\hat{\sigma}_i = -\vec{U}_\infty \cdot \vec{n}_i + U_{n_i} = \frac{\sigma_i}{\left[\beta^2 n_{x_i}^2 + n_{y_i}^2 + n_{z_i}^2 \right]^{1/2}}$$

where $\vec{n}_i = (n_{x_i}, n_{y_i}, n_{z_i})$ is the outward normal of panel i , and U_{n_i} is the normal velocity boundary condition for panel i .

Vortex and Body Doublet Panels

The determination of the vortex and doublet panel singularity strengths is the final step in the solution procedure. They are obtained by solving a set of simultaneous equations utilizing the panel influence equations to relate the singularity strengths to the boundary conditions at control points on the surface. For vortex panels the equation to be satisfied is,

$$\sum_{j=1}^{ntv} A_{ij}^w C_{p_j} + \sum_{j=1}^{ntb} D_{ij}^w \mu_p = -w_{\sigma_i} - w_{\tau_i} - w_{0_i} + (dz_c/dx)_i$$

and for body panels on the internal boundary it is,

$$\sum_{j=1}^{ntv} A_{ij}^\phi C_{p_j} + \sum_{j=1}^{ntb} D_{ij}^\phi \mu_p = -\phi_{\sigma_i} - \phi_{\tau_i} - \phi_{0_i}$$

The known perturbations from others singularities have been placed on the right hand side. Corresponding sets of equations may be written for symmetrical or antisymmetrical loading.

UNIT SOLUTION BOUNDARY CONDITIONS

Several types of basic and unit boundary conditions are considered and can be classified as either symmetric or antisymmetric. Linearized theory allows the superposition of these basic unit solutions. The p, q and r rotary derivative boundary conditions are the result of placing the configuration at $\alpha = 0$, $\beta = 0$ in a flow field rotating at one radian per second.

Symmetric:

$$1a) \text{ Basic - vortex panels} \quad (dz_c/dx) = w_{0B} + w_{0\tau} + w_{0\sigma}$$

$$(dz_c/dx) = \text{surface slope due to twist and camber}$$

$$w_{0B} = \text{normalwash induced by slender body thickness and camber}$$

$$w_{0\tau} = \text{normalwash induced by thickness source panels}$$

$$w_{0\sigma} = \text{normalwash induced by body source panels}$$

$$\sigma = -(\vec{U}_\infty \cdot \vec{n}) + U_n = -(\vec{e}_x \cdot \vec{n}) + U_n = -n_x + U_n$$

$$1b) \text{ Basic - body panels (internal boundary)} \quad -\phi_{0\tau} - \phi_{0\sigma}$$

$$\phi_{0\tau} = \text{velocity potential induced by thickness source panels}$$

$$\phi_{0\sigma} = \text{velocity potential induced by body source panels}$$

$$2a) \quad \text{Unit alpha - vortex panels} \quad - \frac{\pi}{180} \cos \theta_c - w_{\alpha_B} - w_{\alpha_\sigma}$$

$$\theta_c = \text{dihedral angle}$$

$$w_{\alpha_B} = \text{normalwash induced by slender body at unit alpha}$$

$$w_{\alpha_\sigma} = \text{normalwash induced by body source panels at unit alpha}$$

$$\sigma = - (\vec{U}_\infty \cdot \vec{n}) = - \frac{\pi}{180} (\vec{e}_z \cdot \vec{n}) = - \frac{\pi}{180} n_z$$

$$2b) \quad \text{Unit alpha - body panels} \quad - \phi_{\alpha_\sigma}$$

$$\phi_{\alpha_\sigma} = \text{velocity potential induced by body at unit alpha}$$

$$3a) \quad \text{Unit q rotation - vortex panels} \quad - \frac{2}{\dot{c}} (x-x_{cg}) \cos \theta_c - w_{q_B} - w_{q_\sigma}$$

$$w_{q_B} = \text{normalwash induced by slender body undergoing unit q rotation}$$

$$w_{q_\sigma} = \text{normalwash induced by body panels undergoing unit q rotation}$$

$$\sigma = - (\vec{U}_\infty \cdot \vec{n}) = - \frac{2}{\dot{c}} \left[(x-x_{cg}) n_z - (z-z_{cg}) n_x \right]$$

$$3b) \quad \text{Unit q rotation - body panels} \quad - \phi_{q_\sigma}$$

$$\phi_{q_\sigma} = \text{velocity potential induced by body panels undergoing unit q rotation}$$

$$4a) \quad \text{Unit flap - vortex panel} \quad - \frac{\pi}{180} \kappa$$

$$\kappa = 1. \text{ for flap panel}$$

$$\kappa = 0. \text{ for others}$$

$$4b) \quad \text{Unit flap - body panel} \quad 0 \quad \sigma = 0$$

Antisymmetric:

$$1a) \text{ Unit beta - vortex panels} \quad - \frac{\pi}{180} \sin \theta_c - w_{\beta_B} - w_{\beta_\sigma}$$

$$\theta_c = \text{dihedral angle}$$

$$w_{\beta_B} = \text{normalwash induced by slender body at unit sideslip}$$

$$w_{\beta_\sigma} = \text{normalwash induced by body source panels at unit sideslip}$$

$$\sigma = - (\vec{U}_\infty \cdot \vec{n}) = - \frac{\pi}{180} (\vec{e}_y \cdot \vec{n}) = - \frac{\pi}{180} n_y$$

$$1b) \text{ Unit beta - body panels} \quad - \phi_{\beta_\sigma}$$

$$\phi_{\beta_\sigma} = \text{velocity potential induced by body}$$

$$2a) \text{ Unit p rotation - vortex panels}$$

$$- \frac{2}{b} \left[(y-y_{cg}) \cos \theta_c + (z-z_{cg}) \sin \theta_c \right] - w_{p_B} - w_{p_\sigma}$$

$$w_{p_B} = \text{normalwash induced by slender body undergoing unit p rotation}$$

$$w_{p_\sigma} = \text{normalwash induced by body panels undergoing unit p rotation}$$

$$\sigma = - (\vec{U}_\infty \cdot \vec{n}) = - \frac{2}{b} \left[(y-y_{cg}) n_z - (z-z_{cg}) n_y \right]$$

$$2b) \text{ Unit p rotation - body panels} \quad - \phi_{p_\sigma}$$

$$\phi_{p_\sigma} = \text{velocity potential induced by body panels undergoing unit q rotation}$$

$$3a) \quad \text{Unit r rotation - vortex panels} \quad - \frac{2}{b} (x-x_{cg}) \sin \theta_c - w_{r_B} - w_{r_\sigma}$$

w_{r_B} = normalwash induced by slender body
undergoing unit r rotation

w_{r_σ} = normalwash induced by body panels
undergoing unit r rotation

$$\sigma = - (\vec{U}_\infty \cdot \vec{n}) = - \frac{2}{b} \left[(x-x_{cg}) n_y - (y-y_{cg}) n_x \right]$$

$$3b) \quad \text{Unit r rotation - body panels} \quad - \phi_{r_\sigma}$$

ϕ_{r_σ} = velocity potential induced by body
panels undergoing unit r rotation

$$4a) \quad \text{Unit flap - vortex panel} \quad - \frac{\pi}{180} \kappa$$

κ = 1. for flap panel

κ = 0. for others

$$4b) \quad \text{Unit flap - body panel} \quad 0 \quad \sigma = 0$$

CONSTANT SOURCE AND CONSTANT VORTICITY PANEL INFLUENCE EQUATIONS

The source finite elements have a discontinuity in normal velocity across the panel surface while the vortex finite elements have a discontinuity in the tangential velocity in a direction normal to the panel leading edge. The magnitude of the discontinuity, in each case, is constant over the panel area. In addition the vortex panels have a system of trailing vortices extending undeflected to downstream infinity.

A constant pressure or constant source panel with a quadrilateral shape can be constructed (figure 7) by adding or subtracting four semi-infinite triangular shaped panels. These semi-infinite triangles, each determined by a corner of the quadrilateral, can be assumed to induce a velocity perturbation everywhere in the flow. However, each corner represents only an integration limit, and all four corners must be included to make any sense. These perturbation potential expressions are derived in Appendix A.

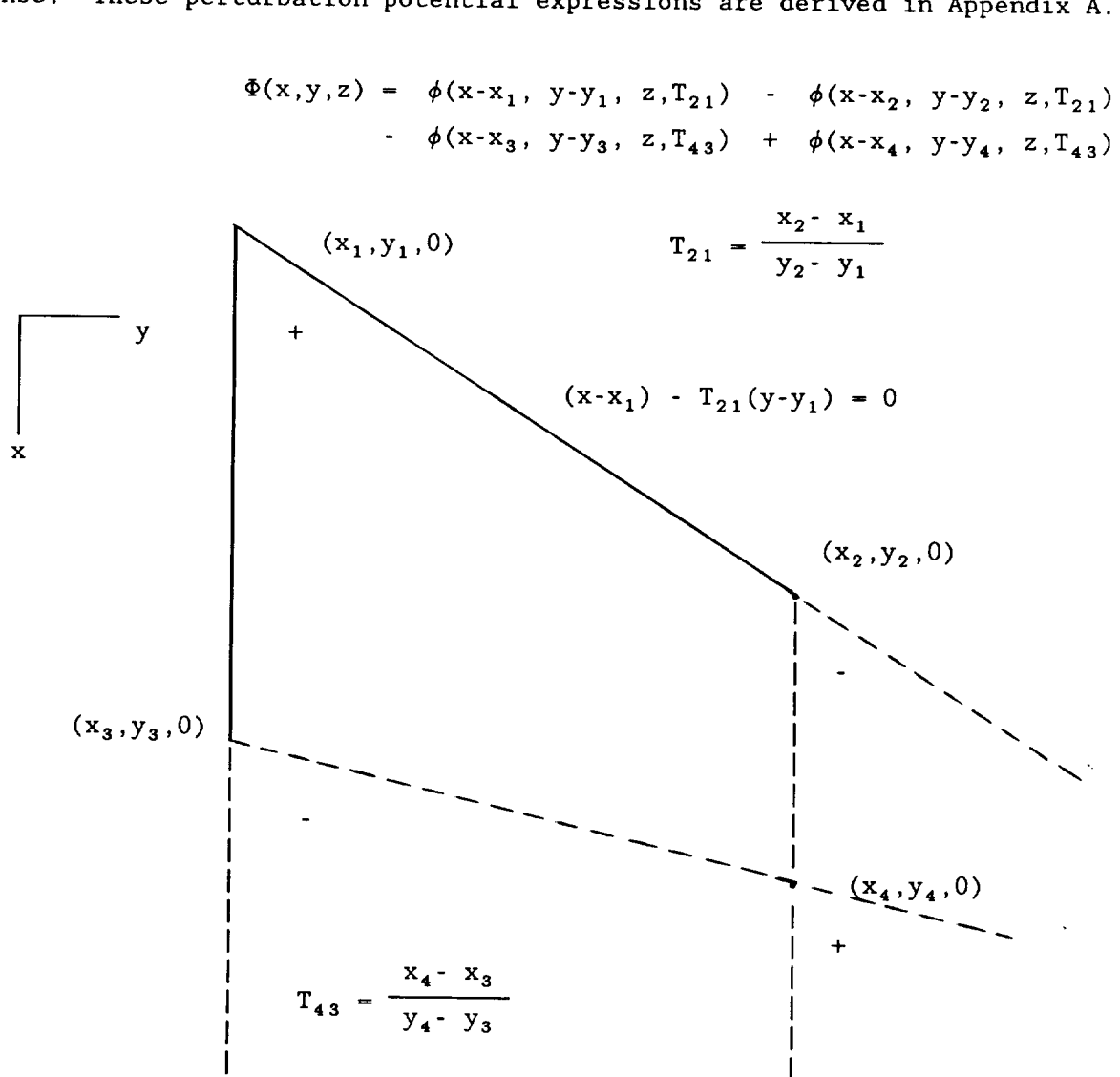


Figure 7. Constant Pressure or Constant Source Panel Construction.

For one corner, having sides determined by $y = 0$ and $x - Ty = 0$:

$$r^2 = y^2 + z^2, \quad R^2 = x^2 + \beta^2 r^2, \quad k = \begin{cases} 1 & \beta^2 > 0 \\ 2 & \beta^2 < 0 \end{cases}, \quad \beta^2 = 1 - M_\infty^2, \quad B^2 = T^2 + \beta^2$$

constant source panel

$$\phi_s(x, y, z, T) = - \frac{\sigma k}{4\pi} \left\{ y \frac{1}{2} \log \frac{R+x}{R-x} + (x-Ty) \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} + z \tan^{-1} \frac{zR}{xy-Tr} \right\}$$

$$u_s(x, y, z, T) = - \frac{\sigma k}{4\pi} \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)}$$

$$v_s(x, y, z, T) = - \frac{\sigma k}{4\pi} \left\{ \frac{1}{2} \log \frac{R+x}{R-x} - T \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} \right\}$$

$$w_s(x, y, z, T) = - \frac{\sigma k}{4\pi} \tan^{-1} \frac{zR}{xy-Tr}$$

constant vorticity panel

$$\phi_v(x, y, z, T) = \frac{kCp}{8\pi} \left\{ Tz \frac{1}{2} \log \frac{R+x}{R-x} - z \frac{B^2}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} + (x-Ty) \tan^{-1} \frac{zR}{xy-Tr} - (2-k) \left[Tz \frac{1}{2} \log r^2 + (x-Ty) \tan^{-1} \frac{y}{z} \right] \right\}$$

$$u_v(x, y, z, T) = \frac{kCp}{8\pi} \left\{ \tan^{-1} \frac{zR}{xy-Tr} - (2-k) \tan^{-1} \frac{y}{z} \right\}$$

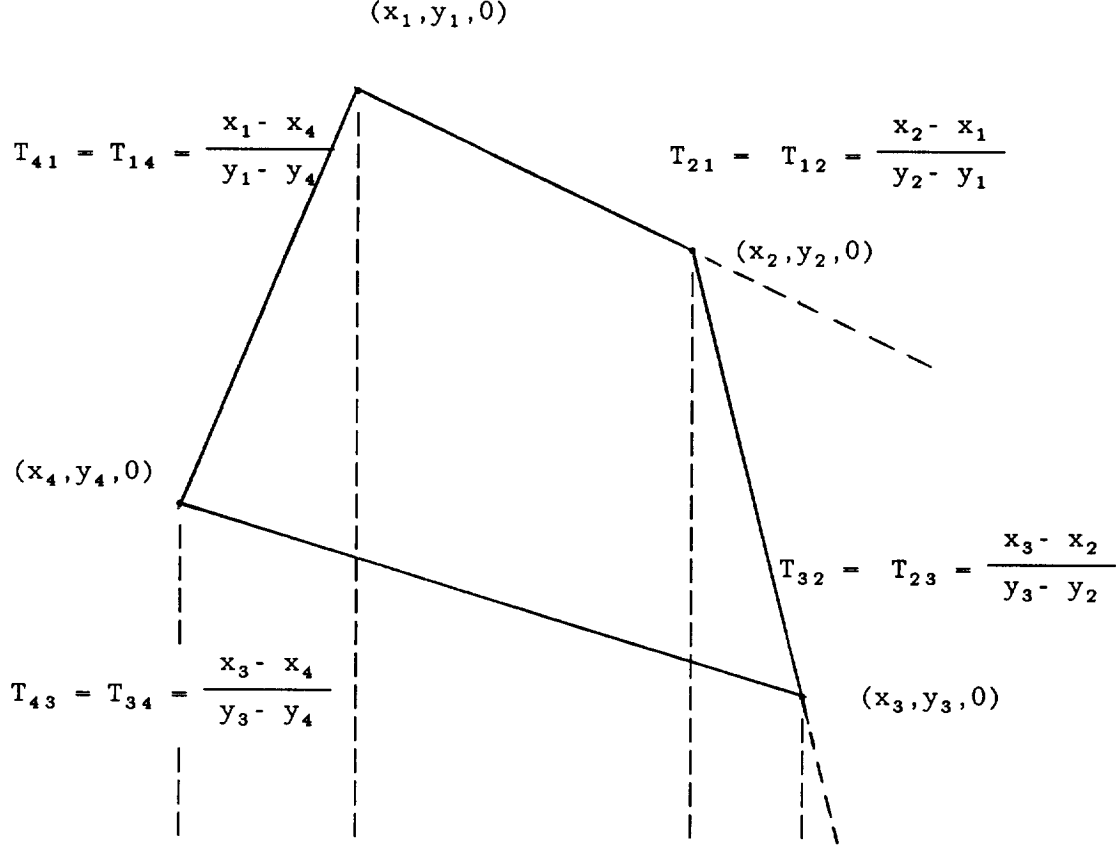
$$v_v(x, y, z, T) = - \frac{kCp}{8\pi} \left\{ T \tan^{-1} \frac{zR}{xy-Tr} + \frac{zR}{r} - (2-k) \left[T \tan^{-1} \frac{y}{z} - \frac{zx}{r} \right] \right\}$$

$$w_v(x, y, z, T) = \frac{kCp}{8\pi} \left\{ T \frac{1}{2} \log \frac{R+x}{R-x} - \frac{B^2}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} + \frac{yR}{r} - (2-k) \left[T \frac{1}{2} \log r^2 - \frac{yx}{r} \right] \right\}$$

Only the real (not imaginary), downstream, contributions are considered when $M_\infty > 1$.

CONSTANT SOURCE AND CONSTANT DOUBLET PANEL INFLUENCE EQUATIONS

Source and vortex panels used to represent body shapes may have an arbitrary quadrilateral shape, i.e. they need not have two streamwise edges. The influence equations may be written in the $z = 0$ plane, and a coordinate transformation used to obtain the perturbations of a panel having arbitrary orientation (see Appendix C). A quadrilateral source panel of arbitrary shape can be constructed by combining quadrilaterals with streamwise parallel sides.



$$\begin{aligned}
 \Phi(x, y, z) &= \phi_s(x-x_1, y-y_1, z, T_{21}) - \phi_s(x-x_2, y-y_2, z, T_{21}) \\
 &+ \phi_s(x-x_2, y-y_2, z, T_{32}) - \phi_s(x-x_3, y-y_3, z, T_{32}) \\
 &- \phi_s(x-x_4, y-y_4, z, T_{34}) + \phi_s(x-x_3, y-y_3, z, T_{34}) \\
 &+ \phi_s(x-x_4, y-y_4, z, T_{41}) - \phi_s(x-x_1, y-y_1, z, T_{41}) \\
 &= \phi_s(x-x_1, y-y_1, z, T_{21}) - \phi_s(x-x_1, y-y_1, z, T_{41}) \\
 &+ \phi_s(x-x_2, y-y_2, z, T_{32}) - \phi_s(x-x_2, y-y_2, z, T_{12}) \\
 &+ \phi_s(x-x_3, y-y_3, z, T_{43}) - \phi_s(x-x_3, y-y_3, z, T_{23}) \\
 &+ \phi_s(x-x_4, y-y_4, z, T_{14}) - \phi_s(x-x_4, y-y_4, z, T_{34})
 \end{aligned}$$

Therefore each corner consists of the difference between the perturbations induced by the two sweep angles. Therefore we can omit terms independent of T , since they will cancel when the two contributions are combined.

Therefore, omitting terms independent of T, the perturbation velocities and perturbation potential for an arbitrary quadrilateral constant source panel are:

Constant source panel

$$\phi_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \left\{ (x-Ty) \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} + z \tan^{-1} \frac{zR}{xy-Tr} \right\}$$

$$u_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)}$$

$$v_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \frac{T}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)}$$

$$w_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \tan^{-1} \frac{zR}{xy-Tr}$$

A constant doublet panel is obtained by taking the z derivative of the constant source panel.

$$\phi_d(x,y,z,T) = -\frac{\mu k}{4\pi} \tan^{-1} \frac{zR}{xy-Tr}$$

$$u_d(x,y,z,T) = -\frac{\mu k}{4\pi} \frac{1}{R} \frac{z (Tx+\beta^2 y)}{[(x-Ty)^2 + B z^2]}$$

$$v_d(x,y,z,T) = -\frac{\mu k}{4\pi} \frac{1}{R} \frac{Tz (Tx+\beta^2 y)}{[(x-Ty)^2 + B z^2]}$$

$$w_d(x,y,z,T) = -\frac{\mu k}{4\pi} \frac{1}{R} \frac{(x-Ty) (Tx+\beta^2 y)}{[(x-Ty)^2 + B z^2]}$$

Since the sweep angle could become infinite, we can write the above equations in a different form. First, the sweep angle can be written as

$$T = \frac{\Delta X}{\Delta Y}, \quad \text{then define } \hat{B}^2 = \Delta X^2 + \beta^2 \Delta Y^2$$

where e.g. for $T = T_{12}$ $\Delta X = x_2 - x_1$ $\Delta Y = y_2 - y_1$

Now for the source panel, using the previous definitions, we can write,

$$\phi_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \left\{ (x \Delta Y - y \Delta X) \frac{1}{\hat{B}} \frac{1}{2} \log \frac{\hat{B}R + (x \Delta X + \beta^2 y \Delta Y)}{\hat{B}R - (x \Delta X + \beta^2 y \Delta Y)} + z \tan^{-1} \frac{\Delta Y z R}{xy \Delta Y - r^2 \Delta X} \right\}$$

$$u_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \frac{\Delta Y}{\hat{B}} \frac{1}{2} \log \frac{\hat{B}R + (x \Delta X + \beta^2 y \Delta Y)}{\hat{B}R - (x \Delta X + \beta^2 y \Delta Y)}$$

$$v_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \frac{\Delta X}{\hat{B}} \frac{1}{2} \log \frac{\hat{B}R + (x \Delta X + \beta^2 y \Delta Y)}{\hat{B}R - (x \Delta X + \beta^2 y \Delta Y)}$$

$$w_s(x,y,z,T) = -\frac{\sigma k}{4\pi} \tan^{-1} \frac{\Delta Y z R}{xy \Delta Y - r^2 \Delta X}$$

The constant doublet panel is now.

$$\phi_d(x,y,z,T) = -\frac{\mu k}{4\pi} \tan^{-1} \frac{\Delta Y z R}{xy \Delta Y - r^2 \Delta X}$$

$$u_d(x,y,z,T) = -\frac{\mu k}{4\pi} \frac{1}{R} \frac{z \Delta Y (x \Delta X + \beta^2 y \Delta Y)}{[(x \Delta Y - y \Delta X)^2 + \hat{B}^2 z^2 \Delta Y^2]}$$

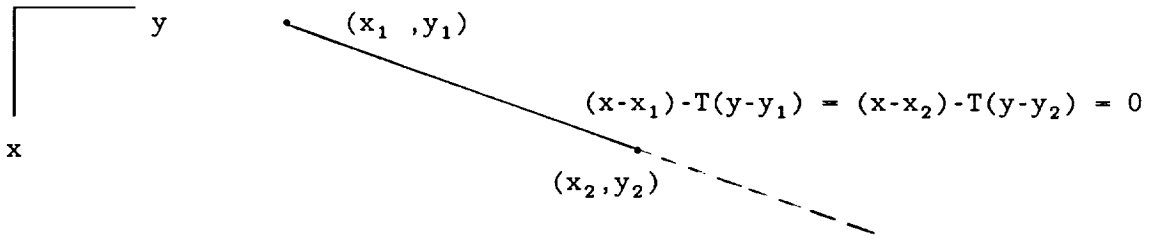
$$v_d(x,y,z,T) = -\frac{\mu k}{4\pi} \frac{1}{R} \frac{z \Delta X (x \Delta X + \beta^2 y \Delta Y)}{[(x \Delta Y - y \Delta X)^2 + \hat{B}^2 z^2 \Delta Y^2]}$$

$$w_d(x,y,z,T) = -\frac{\mu k}{4\pi} \frac{1}{R} \frac{(x \Delta Y - y \Delta X) (x \Delta X + \beta^2 y \Delta Y)}{[(x \Delta Y - y \Delta X)^2 + \hat{B}^2 z^2 \Delta Y^2]}$$

LINEARLY VARYING SOURCE PANEL INFLUENCE EQUATIONS

In supersonic flow constant source panels having a sonic edge have a real singularity along an extension of this edge. The singularity occurs because:

$$\lim_{\substack{(x-x_1) - T(y-y_1) \rightarrow 0 \\ \epsilon^2 = (T^2 + \beta^2) \rightarrow 0}} \frac{1}{\epsilon} \left\{ \frac{1}{2} \log \frac{\epsilon R_1 + [T(x-x_1) + \beta^2 (y-y_1)]}{\epsilon R_1 - [T(x-x_1) + \beta^2 (y-y_1)]} - \frac{1}{2} \log \frac{\epsilon R_2 + [T(x-x_2) + \beta^2 (y-y_2)]}{\epsilon R_2 - [T(x-x_2) + \beta^2 (y-y_2)]} \right\} = \infty$$



Control points which are near the extension of this edge will have large u and v velocities induced upon them. The singularity can be eliminated by using panels which have a source distribution which varies linearly in the chordwise direction. The resulting continuous source distribution eliminates the singularities. The linearly varying source panel influence equations can be found by integrating the constant source panel influence with respect to x .

$$u_{10} = \frac{-k}{2\pi} \left\{ y \frac{1}{2} \log \frac{R+x}{R-x} + (x-Ty) \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} + z \tan^{-1} \frac{zR}{xy - Tr} \right\}$$

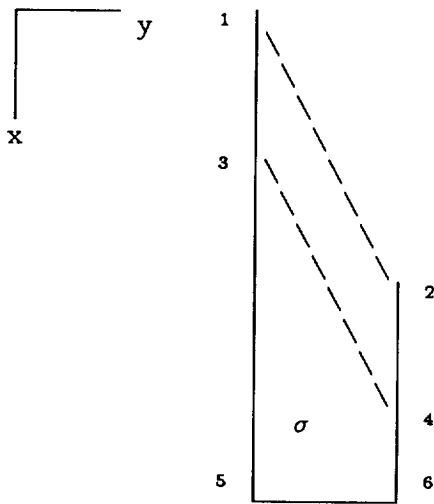
$$v_{10} = \frac{-k}{2\pi} \left\{ (x-Ty) \frac{1}{2} \log \frac{R+x}{R-x} - T(x-Ty) \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} - Tz \tan^{-1} \frac{zR}{xy - Tr} - R \right\}$$

$$w_{10} = \frac{-k}{2\pi} \left\{ Tz \frac{1}{2} \log \frac{R+x}{R-x} - B^2 z \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} + (x-Ty) \tan^{-1} \frac{zR}{xy - Tr} \right\}$$

$$\phi_{10} = \frac{-k}{2\pi} \left\{ Tz \frac{1}{2} \log \frac{R+x}{R-x} - B^2 z \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} + (x-Ty) \tan^{-1} \frac{zR}{xy - Tr} \right\}$$

These velocity components satisfy the same criteria as the velocity components for the constant source panels except that the source strength is proportional to $(x-Ty)$. The source panel finite elements are constructed with the following properties.

1. All panel leading and trailing edges are at constant (x/c) , side edges are at constant y .
2. Each source finite element is composed of a pair of chordwise adjacent panels.
3. The source strength varies linearly with chord measured from the leading edge of a panel pair, i.e. the maximum value of the source strength is proportional to the local chord and attains this maximum on the panel edge joining the panel pair.



$$\Delta_{31} = (x/c)_3 - (x/c)_1 = (x/c)_4 - (x/c)_2$$

$$\Delta_{53} = (x/c)_5 - (x/c)_3 = (x/c)_6 - (x/c)_4$$

The perturbation velocities induced by this panel pair are composed of contributions from six corners.

$$\begin{aligned}
 u(x,y,z) = & \sigma/\Delta_{31} [u_{10}(x-x_1, y-y_1, z, T_1) - u_{10}(x-x_3, y-y_3, z, T_3) \\
 & - u_{10}(x-x_2, y-y_2, z, T_2) - u_{10}(x-x_4, y-y_4, z, T_4)] \\
 & + \sigma/\Delta_{53} [u_{10}(x-x_5, y-y_5, z, T_5) - u_{10}(x-x_3, y-y_3, z, T_3) \\
 & - u_{10}(x-x_6, y-y_6, z, T_6) - u_{10}(x-x_4, y-y_4, z, T_4)]
 \end{aligned}$$

If there are N panels in the chordwise direction there will be $N-1$ singularities or unknown source strengths associated with them. The linear variation in the source distribution means the value of dz/dx must be zero at the leading and trailing edges of each span station. This may be an undesired restriction and therefore the use of linearly varying source panels is optional.

EDGE EFFECTS

The low pressure created by high velocities around a surface subsonic leading edge results in a suction force. As the edge becomes thinner or the angle of attack increases, the flow deviates from potential conditions resulting in a progressive loss of theoretical suction and an increase in drag. Generalizing a concept due to Polhamus⁵, it is assumed that the leading edge vortex created by the detached flow in effect rotates the lost suction force perpendicular to the local surface.

In order to implement this philosophy, a method of determining the spanwise variation of potential suction was developed using linear thin wing

theory and involves finding the coefficient of the $1/\sqrt{x}$ term in the chordwise net pressure distribution. The analysis is applicable to multiple surface problems of arbitrary planform in the presence of bodies at any Mach number. If the chordwise net pressure distribution on a thin wing at any given span station is expanded in a series

$$\Delta C_p = A_0 \cot\left(\frac{\phi}{2}\right) + \sum_{n=1}^N A_n \sin(n\phi)$$

$$\xi = x/c = \frac{1}{2} (1 - \cos \phi) = \sin^2\left(\frac{\phi}{2}\right)$$
(12)

it is shown in appendix B that the leading edge nondimensionalized suction force per unit length is

$$C_s(y) = \frac{\Delta \text{THRUST}}{c \Delta y q_\infty} = \frac{\pi}{8} \frac{A_0^2}{\sqrt{T + \beta^2}} \quad (13)$$

where $T = \tan^2 \Lambda_{L.E}$; $\beta^2 = 1 - M_\infty^2$

and c is the local chord.

Only the first term in equation 12 contributes to the thrust, since it is the only one which is infinite at the leading edge. If the chordwise pressures are known at M points along the chord, the coefficients A_n are obtained by fitting a least square error curve described by N terms of the series, through the points, where $N < M$. The pressure distribution is obtained using constant pressure panel analysis.

The method used to compute the suction force at surface tips is similar to that for the leading edge. By using the irrotational property of the flow, it is shown in appendix B that the tip suction force is

$$C_s(\zeta) = \Delta F_\eta / (q_\infty c_T \Delta x) = (\pi/32) [c_{AVG}^2 / (c_T \eta_{MAX})] c_{n_0}^2 \left[\int_0^\zeta f(x) dx \right]^2 \quad (14)$$

where

c_{AVG}	surface average chord
c_T	tip chord
η_{MAX}	tip surface lateral surface dimension
ζ, x	fraction of chord
c_{n_0}	$c_n(\eta) [c(\eta)/c_{AVG}] \eta_{MAX} [(\eta_{MAX}^2 - \eta^2)/2]^{-1/2}$
c_n	is local section normal force coefficient

and as $\eta \rightarrow \eta_{MAX}$ the net pressure coefficient is assumed to be of the form

$$\Delta C_p(\zeta, \eta) = \left\{ [1 - (\eta/\eta_{MAX})^2] / 2 \right\}^{1/2} (c_{AVG}/c(\eta)) c_{n_0} f(\zeta)$$

The sectional leading edge suction attained in the real flow, $\tilde{C}_s(y)$, is estimated by⁶

$$\tilde{C}_s(y) = K_s(y) C_s(y)$$

where

$$K_s(y) = 2 M_e^{-1} (1 - M_e^2) [(t_n/c_n) (r_n/c_n)^{0.4} / (\beta_n C_{s,n})]^{0.6} \leq 1$$

and

$$M_e = -\sqrt{2} \bar{C}_p^{-1} [(1+\bar{C}_p^2)^{1/2} - 1]^{1/2}$$

$$\bar{C}_p = \gamma \beta_n C_{p,LIM} ; \beta_n = (1-M_n^2)^{1/2}$$

$$C_{p,LIM} = -2/(\gamma M_n^2) [(R_n \times 10^6)/(R_n \times 10^{-6} + 10^{(4-3M_n^2)})] (.05 + .35(1-M_n^2)^2)$$

$$R_n = R_{\bar{c}} (c_n/\bar{c}) \cos(\Lambda_{LE})$$

$$C_{s,n} = C_s (c/c_n) / \cos^2(\Lambda_{LE})$$

$$M_n = M_\infty \cos(\Lambda_{LE})$$

The chord of the normal section, c_n , is defined so as to place the maximum thickness, t_n , at the mid-chord as indicated in figure 8. The associated leading edge radius is designated by r_n

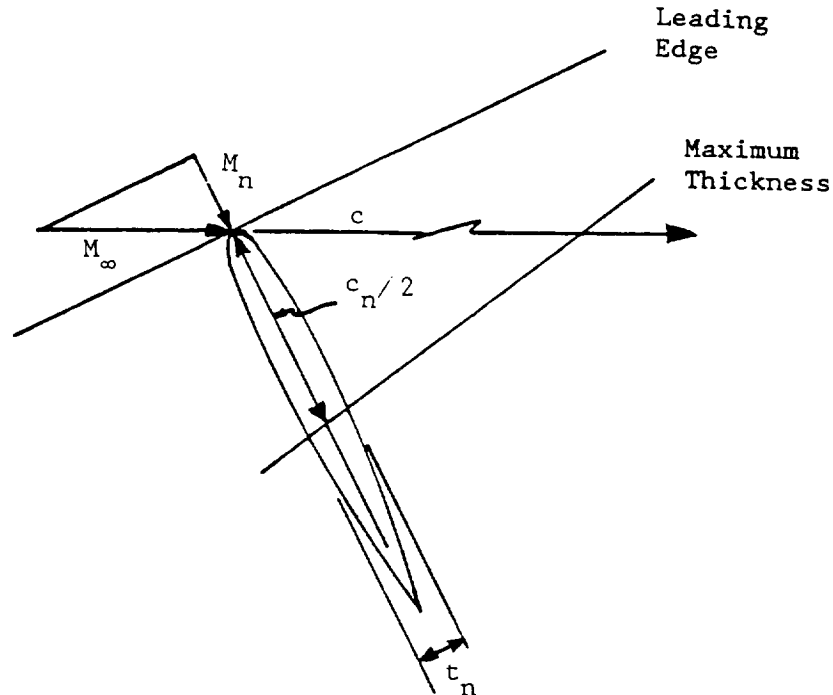


Figure 8. Definition of Normal Section Characteristics.

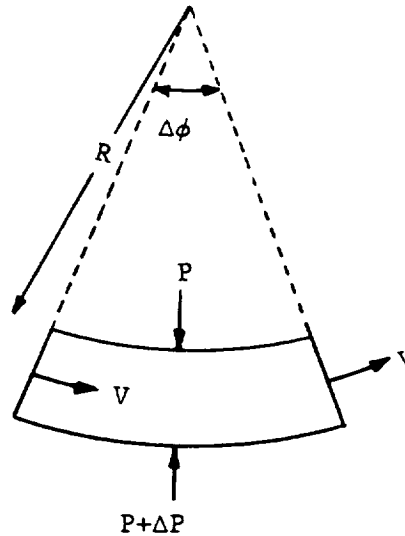
Potential tip suction is assumed to be fully rotated as a result of vortex formation in the present analysis.

JET FLAP

A completely linearized approach is used based on the assumptions of thin airfoil theory. The flow is assumed to be inviscid and irrotational and all entrainment effects are neglected. The jet is represented by an infinitesimally thin sheet having zero mass flow but finite momentum per unit of span. This sheet is assumed to extend from the trailing edge of the surface back to infinity. (In practice one or two chord lengths is sufficient). The effects of transverse momentum and the deflection of the jet sheet are neglected.

Since both the planform and jet can maintain a pressure discontinuity, they are both represented by a system of quadrilateral panels having continuous distributions of vorticity. The strengths of these constant pressure vortex panels are determined by solving a set of linear simultaneous equations which satisfy the downwash boundary conditions at a set of control points on the planform and jet.

The boundary condition on the planform is the previously described flow tangency condition. The pressure difference across the jet causes a change in the direction of the jet momentum. The equation relating these quantities forms the boundary condition on the jet and can be derived by considering a jet segment of unit depth.



The mass rate of flow through the jet is \dot{m} and the velocity is V. If we assume a pressure difference of ΔP across the jet, then from the momentum theorem applied to the differential element, we write

$$\dot{m} V_j \Delta\phi = R \Delta P \Delta\phi$$

or

$$\Delta P = \dot{m} V_j / R$$

where R is the radius of curvature of the jet.

The reaction of the jet on the flow external to the jet is

$$F = R \Delta P \Delta \phi$$

A vortex of strength γ per unit length along the jet would produce a reaction of

$$F = \rho U_{\infty} \gamma R \Delta \phi$$

Hence, equating these two forces, we calculate the action of the jet on the flow external to the jet by replacing the jet with a running vortex strength γ given by

$$\gamma = \dot{m} V_j / (\rho U_{\infty} R)$$

For a nearly horizontal jet with a large radius of curvature

$$1/R \approx \frac{d^2 z}{dx^2} = \frac{dw}{dx}$$

where w is local downwash velocity (nondimensionalized with respect to U_{∞}).

Then

$$\frac{1}{2} \Delta C_p = \frac{\gamma}{U_{\infty}} = \frac{1}{2} \rho U_{\infty}^2 C \frac{\dot{m} V_j}{q C} \frac{1}{\rho U_{\infty}} \frac{dw}{dx}$$

or
$$C_{\mu}(y) C(y) \frac{\partial}{\partial x} w(x,y) - \Delta C_p(x,y) = 0$$

which is the boundary condition written for a three dimensional jet flap.

To apply the jet flap boundary condition to control point i , the above equation is integrated between adjacent control points in the streamwise direction.

$$C_{\mu}(y) C(y) \int_{x_{c_{i-1}}}^{x_{c_i}} \frac{\partial}{\partial x} w(x,y) dx - \int_{x_{c_{i-1}}}^{x_{c_i}} \Delta C_p(x,y) dx = 0 \quad (15)$$

The control point is located at 87.5 percent of each panel chord. To simplify the second integral in equation (15), the assumption is made that the control point is exactly at the panel trailing edge. The effects of such an assumption have been shown to be negligible. Equation (15) evaluated from the leading to the trailing edge of panel i yields the following relation:

$$C_{\mu} C [w_i - w_{i-1}] - \Delta C_{p_i} \Delta x_i = 0 \quad (16)$$

The downwash at each control point is written in terms of the N net pressures on the quadrilateral panels:

$$w_i = \sum_{j=1}^N A_{ij} \Delta C_{P_j}$$

$$w_{i-1} = \sum_{j=1}^N A_{i-1j} \Delta C_{P_j}$$

Equation (16) is then written

$$\sum_{j=1}^N \left\{ C_{\mu} C [A_{ij} - A_{i-1j}] - \delta_{ij} \Delta x_i \right\} \Delta C_{P_j} = 0$$

where δ_{ij} is the Kroneker delta.

For a flap panel adjacent to the jet exit, equation (16) must include any jet deflection angle relative to the surface trailing edge.

$$C_{\mu} C [w_i - (w_{i-1} + \delta_j)] - \Delta C_{P_i} \Delta x_i = 0$$

or

$$C_{\mu} C [w_i - w_{i-1}] - \Delta C_{P_i} \Delta x_i = C_{\mu} C \delta_j$$

where δ_j is the jet deflection angle.

Then

$$\sum_{j=1}^N \left\{ C_{\mu} C [A_{ij} - A_{i-1j}] - \delta_{ij} \Delta x_i \right\} \Delta C_{P_j} = C_{\mu} C \delta_j$$

The complete set of linear simultaneous equations for both the surface and the jet flap is then written

$$\sum_{j=1}^N \tilde{A}_{ij} \Delta C_{Pj} = C_i ; \quad i = 1, N \quad (17)$$

where

$$\tilde{A}_{ij} = \begin{cases} A_{ij} & \text{for } i \text{ on the surface} \\ C_{\mu} C [A_{ij} - A_{i-1j}] - \delta_{ij} \Delta x_i & \text{for } i \text{ on the jet} \end{cases}$$

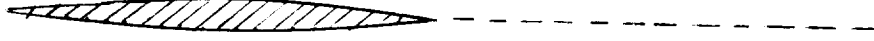
$$C_i = \begin{cases} -\alpha_i & \text{for } i \text{ on the surface} \\ C_{\mu} C \delta_i & \text{for } i \text{ on the jet adjacent to the exit} \\ 0 & \text{for } i \text{ elsewhere on the jet} \end{cases}$$

Both symmetric and antisymmetric jet deflections are considered. Thus, after calculating the influence matrices and boundary conditions in the usual manner, the appropriate rows are modified and combined to produce a linear symmetric or antisymmetric system as described by equation (17). Because of the rotational quality of the flow fields, the p, q and r rotary derivative calculations are generally not valid for jet flap configurations.

INLETS

A jet boundary between two flows with different total energies is characterized by a discontinuity in tangential velocity, but a continuous value of C_p . This flow can be replaced by a flow with the same total energy everywhere, but now having a discontinuity in C_p across the jet boundary, instead of a discontinuity in total energy. The jet boundary will be represented by a vortex sheet having the same discontinuity in tangential velocity, and the velocities will be the same everywhere in the two flows. If the jet is such that the perturbation velocities are small compared with the free stream velocity, i.e. $u, v, w \ll U_{\infty}$, this jet boundary can be simulated by constant ΔC_p chord plane panels. It will be shown that, to first order, this value of ΔC_p is constant on the entire jet boundary. Let u be the x component of the perturbation velocity. Then, assuming energy addition to the jet flow, write the energy equation across the jet boundary:

$$\frac{\rho}{\rho_{\infty}} [(1+u)^2 + v^2 + w^2] + \frac{1}{2} \frac{\Delta H}{\rho_{\infty} U_{\infty}^2} = \frac{\rho_j}{\rho_{\infty}} [(1+u_j)^2 + v_j^2 + w_j^2]$$



$$\begin{array}{c} \text{inside} \quad \rho_j, U_j = U_{\infty}(1+u_j) \\ \text{outside} \quad \rho, U = U_{\infty}(1+u) \end{array}$$

To first order, $\frac{\rho}{\rho_{\infty}} = 1 - M_{\infty}^2 u$

therefore

$$[1 - M_{\infty}^2 u] [(1+u)^2 + v^2 + w^2] + \frac{1}{2} \frac{\Delta H}{\rho_{\infty} U_{\infty}^2} = [1 - M_{\infty}^2 u_j] [(1+u_j)^2 + v_j^2 + w_j^2]$$

to first order the energy equation becomes,

$$[1 + (2-M_{\infty}^2)u] + \frac{1}{2} \frac{\Delta H}{\rho_{\infty} U_{\infty}^2} = [1 + (2-M_{\infty}^2)u_j]$$

therefore $(1-M_{\infty}^2)(u_j - u) = \frac{1}{2} \frac{\Delta H}{\rho_{\infty} U_{\infty}^2} = \frac{1}{2} \Delta C_p$

or $\Delta C_p = -2(u_j - u) = -\frac{2}{(2-M_{\infty}^2)} \frac{1}{2} \frac{\Delta H}{\rho_{\infty} U_{\infty}^2} = \text{const}$

where ΔC_p is in the direction of the normal pointing into the jet flow

The inlet is simulated by specifying an average mass flux over a set of field points within the inlet region. The nondimensional mass flux per unit area in the x direction can be written:

$$\frac{\rho}{\rho_{\infty}} \frac{U}{U_{\infty}} = (1 - M_{\infty}^2 u) (1 + u) = [1 + (1-M_{\infty}^2)u]$$

The value of this expression can be calculated, on each field point, for a unit value of ΔC_p across each panel of the configuration and jet boundary. Therefore a linear equation can be written to constrain the value of the average of the mass flux, and therefore the inlet mass flux, on a given set of field points. The additional unknown required for the additional equation is supplied by the (constant) value of ΔC_p across the jet boundary.

AERODYNAMIC CHARACTERISTICS

Longitudinal and lateral-directional forces and moments due to thickness, \hat{a} twist and \hat{a} camber, pitch, sideslip, and the dimensionless rotary velocities \hat{p} , \hat{q} , and \hat{r} are obtained from surface pressure integrations of the various configuration components.

Slender Bodies

The pressure coefficient, to an approximation consistent with slender body theory, is

$$C_p = \frac{(P - P_\infty)}{q_\infty} = -2 \left\{ \varphi_x + g'(x) + \left[\alpha + \hat{q} \frac{(x - x_{CG})}{c/2} + \hat{p} \frac{y}{b/2} \right] \phi_z - \left[\beta + \hat{r} \frac{(x - x_{CG})}{b/2} + \hat{p} \frac{(z - z_{CG})}{b/2} \right] \phi_y \right\} - \phi_y^2 - \phi_z^2 \quad (18)$$

Paneled Bodies

For paneled bodies a surface differentiation of the perturbation potential is used to obtain the perturbation velocity components tangential to the surface. The velocity normal to the surface is obtained from the imposed boundary condition. A formula for the pressure coefficient can be derived using the energy equation. Although the perturbation velocities were obtained using a linear equation, on body surfaces, which may be quite thick, a better approximation to the pressure coefficient is obtained using a nonlinear formula. Assuming the freestream is at an angle of attack α and an angle of sideslip β , and the perturbation velocities are nondimensional, we can write for the freestream,

$$\vec{U} = U_\infty \left[\cos \alpha \cos \beta \vec{e}_x - \cos \alpha \sin \beta \vec{e}_y + \sin \alpha \vec{e}_z \right]$$

For the energy equation we can write,

$$\frac{\gamma R}{\gamma - 1} T + \frac{1}{2} U_\infty^2 \left[(u + \cos \alpha \cos \beta)^2 + (v - \cos \alpha \sin \beta)^2 + (w + \sin \alpha)^2 \right] = \frac{\gamma R}{\gamma - 1} T_\infty + \frac{1}{2} U_\infty^2$$

and since $M_\infty^2 = \frac{U_\infty^2}{\gamma R T_\infty}$

$$1 + \frac{\gamma - 1}{2} M_\infty^2 \left[1 - (u + \cos \alpha \cos \beta)^2 - (v - \cos \alpha \sin \beta)^2 - (w + \sin \alpha)^2 \right] = \frac{T}{T_\infty}$$

$$= 1 - \frac{\gamma-1}{2} M_{\infty}^2 \left[2 (u \cos \alpha \cos \beta - v \cos \alpha \sin \beta + w \sin \alpha) + u^2 + v^2 + w^2 \right]$$

Since the existence of a velocity potential assumes isentropic flow, we can use the isentropic relationship between pressure and temperature.

$$\frac{p}{p_{\infty}} = \left[\frac{T}{T_{\infty}} \right]^{\frac{\gamma}{\gamma-1}} = \left\{ 1 - \frac{\gamma-1}{2} M_{\infty}^2 \left[2 (u \cos \alpha \cos \beta - v \cos \alpha \sin \beta + w \sin \alpha) + u^2 + v^2 + w^2 \right] \right\}^{\frac{\gamma}{\gamma-1}}$$

for small values of α , and β this becomes

$$\frac{p}{p_{\infty}} = \left\{ 1 - \frac{\gamma-1}{2} M_{\infty}^2 \left[2 (u - \beta v + \alpha w) + u^2 + v^2 + w^2 \right] \right\}^{\frac{\gamma}{\gamma-1}}$$

or

$$\frac{p}{p_{\infty}} = \left\{ 1 - \frac{\gamma-1}{2} M_{\infty}^2 \left[2u + u^2 + (v - \beta)^2 - \beta^2 + (w + \alpha)^2 - \alpha^2 \right] \right\}^{\frac{\gamma}{\gamma-1}}$$

Using the above expression the isentropic pressure formula for c_p is:

$$c_p = \frac{p - p_{\infty}}{\frac{1}{2} \gamma p_{\infty} M_{\infty}^2} = \frac{1}{\frac{1}{2} \gamma M_{\infty}^2} \left[\frac{p}{p_{\infty}} - 1 \right]$$

For small values of the quantity δ ,

$$\delta = \frac{\gamma-1}{2} M_{\infty}^2 \left[2 (u - \beta v + \alpha w) + u^2 + v^2 + w^2 \right]$$

we can expand the exponentiation in an infinite series in δ .

$$\frac{p}{p_{\infty}} - 1 = \left[1 - \delta \right]^{\frac{\gamma}{\gamma-1}} - 1 = \frac{\gamma}{\gamma-1} \left[-\delta + \frac{1}{2} \frac{1}{\gamma-1} \delta^2 - \dots \right]$$

Retaining only terms up to order 2 in α , β , u , v , w , we have,

$$\frac{p}{p_\infty} - 1 = -\frac{\gamma}{2} M_\infty^2 \left[2(u - \beta v + \alpha w) + (1 - M_\infty^2) u^2 + v^2 + w^2 \right]$$

or

$$\begin{aligned} c_p &= \frac{p - p_\infty}{\frac{1}{2} \gamma p_\infty M_\infty^2} = - \left[2(u - \beta v + \alpha w) + (1 - M_\infty^2) u^2 + v^2 + w^2 \right] \\ &= - \left[2u + (1 - M_\infty^2) u^2 + (v - \beta)^2 - \beta^2 + (w + \alpha)^2 - \alpha^2 \right] \end{aligned}$$

Where, to first order, the freestream is represented by,

$$\vec{U} = U_\infty \left[\vec{e}_x - \beta \vec{e}_y + \alpha \vec{e}_z \right]$$

Planar Components

Surface pressure distributions are calculated for planar components using the first-order linearized form

$$C_p = -2u/U = -2[(u/U)_{IND} \pm C_{p_{NET}}/4]$$

The \pm signs refer to the upper and lower surfaces respectively. The term $(u/U)_{IND}$ consists of the velocities induced by the isolated bodies and other vortex and source panels. These velocities are obtained by multiplying the (u/U) influence matrices by the appropriate panel strengths. The $C_{p_{NET}}/4$ term accounts for the u/U perturbation velocity induced by the local distribution of vorticity and changes sign from upper to lower surface. The total (u/U) and $C_{p_{NET}}$ values are the result of taking linear combinations of all the basic and unit solutions.

FORCES AND MOMENTS

Slender Bodies

The forces and moments are obtained from the surface integrations. Let ds be an element of arc length at a given x section, and let x be nondimensionalized with respect to the body length L . First performing line integrals at each section and then integrating over x gives,

$$\frac{F_x}{q_\infty L^2} = \int_0^1 \oint C_p (\partial \nu / \partial x) ds dx$$

$$\frac{F_y}{q_\infty L^2} = - \int_0^1 \oint C_p dz dx$$

$$\frac{F_z}{q_\infty L^2} = - \int_0^1 \oint C_p dy dx$$

$$\frac{M_x}{q_\infty L^3} = - \int_0^1 \oint (z - z_{CG}) C_p dz dx - \int_0^1 \oint y C_p dy dx$$

$$\frac{M_y}{q_\infty L^3} = - \int_0^1 (x - x_{CG}) \oint C_p dy dx$$

$$\frac{M_z}{q_\infty L^3} = - \int_0^1 (x - x_{CG}) \oint C_p dz dx$$

In terms of these expressions, the commonly used aerodynamic coefficients are

$$C_x = \frac{F_x}{q_\infty L^2} \frac{L^2}{S_{REF}}$$

$$C_y = \frac{F_y}{q_\infty L^2} \frac{L^2}{S_{REF}}$$

$$C_z = \frac{F_z}{q_\infty L^2} \frac{L^2}{S_{REF}}$$

$$C_\ell = \frac{M_x}{q_\infty L^3} \frac{L^3}{b S_{REF}}$$

$$C_m = \frac{M_y}{q_\infty L^3} \frac{L^3}{\bar{c} S_{REF}}$$

$$C_n = \frac{M_z}{q_\infty L^3} \frac{L^3}{b S_{REF}}$$

where L is the body length and \bar{c} , b and S_{REF} are configuration reference chord, span and area, respectively.

Cross-coupling between the pitch, sideslip, and rotary motions through the product and quadratic terms in equation (18) is neglected.

Paneled Bodies

A surface differentiation done on the perturbation potential for each of the basic and unit solutions to obtain the velocity components tangential to the surface. The velocity normal to the surface is obtained from the imposed boundary condition. These unit solution surface velocities are combined to obtain the resultant pressure coefficient. To obtain the section forces and moments, component forces and moments, and configuration forces and moments, a surface integration is done. Each computed pressure coefficient is multiplied by the panel area and the proper component of the surface normal, and the result is summed over all of the body panels.

Planar Components

The perturbation velocities for each of the basic and unit solutions are combined to give the resultant pressure coefficient. The net pressures and pressure coefficients are then integrated numerically to give the section forces and moments, component forces and moments, and configuration forces and moments.

Since the vortex panels have a constant pressure distribution, a block integration scheme is employed. With the exception of drag, these basic and unit force and moment coefficients are combined in a linear manner to produce the aerodynamic characteristics for any desired flight condition. Since drag varies in a parabolic manner, it must be considered on a point by point basis as defined in a later section.

The longitudinal normal force distribution on the bodies is calculated for each solution. The load distribution on the interference shell portion of the body is given by integrating over all vortex panels at a given longitudinal station.

Normal Force

$$C_n \frac{w}{\ell} = \frac{C_1}{L \Delta x} \sum_{i=1}^N C_{P_{NET_i}} A_i \cos(\theta_i)$$

where N is the number of panels around the shell, L is the length of the body, Δx is the length of the interference shell segment, A_i is the panel area, and $C_1 = 2$ for a centerline body or $C_1 = 1$ for an off centerline body. This carryover load distribution is added to the previously calculated isolated body longitudinal load distribution.

The section characteristics of planar components are determined by a chordwise summation of panel data at each span station and are given by the following equations:

Local Normal Force:

$$C_n = \frac{1}{c \Delta s} \sum_{i=1}^{Nc} C_{P_{NET_i}} A_i$$

Weighted Normal Force:

$$C_n \frac{c}{c_{AVG}} = \frac{1}{c_{AVG} \Delta s} \sum_{i=1}^{Nc} C_{P_{NET_i}} A_i$$

Weighted Lift Force:

$$C_l \frac{c}{c_{AVG}} = \frac{1}{c_{AVG} \Delta s} \sum_{i=1}^{Nc} C_{P_{NET_i}} A_i \cos(\theta_i)$$

Center Of Pressure

$$X_{c.p.} = \frac{1}{c_n c \Delta s} \sum_{i=1}^{Nc} C_{P_{NET_i}} A_i (x_i - x_{LE})$$

where Nc is the number of chordwise panels, θ is the section dihedral angle and Δs is the width of the span station and is given by

$$\Delta s = [\Delta y^2 + \Delta z^2]^{1/2}$$

The section characteristics due to the reaction of a jet flap are calculated by taking the appropriate component of the reaction force.

Reaction Normal Force:

$$C_n \frac{C}{C_{AVG}} = C_\mu \frac{C}{C_{AVG}} \sin \delta_{jT} \approx C_\mu \frac{C}{C_{AVG}} \delta_{jT}$$

where δ_{jT} is the total deflection angle of the jet.

Reaction Lift Force:

$$C_l \frac{C}{C_{AVG}} = C_n \frac{C}{C_{AVG}} \cos \theta$$

Component forces and moments including edge vortex effects are given by the following equations:

Lift:

$$C_{L S_{REF}} = F_1 \sum_{i=1}^N C_{P_{NET_i}} A_i \cos(\theta_i) + \sum_{j=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s'_j \Omega_{L_j} \\ + C_T^2 \sum_{k=1}^{NC_T} C_{s_k} \Delta(x/C_T)_k T_{L_k}$$

Side Force:

$$C_{Y S_{REF}} = F_1 \sum_{i=1}^N C_{P_{NET_i}} A_i \sin(\theta_i) - \sum_{j=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s'_j \Omega_{Y_j} \\ - C_T^2 \sum_{k=1}^{NC_T} C_{s_k} \Delta(x/C_T)_k T_{Y_k}$$

Rolling Moment:

$$C_{\ell b S_{REF}} = -F_2 \sum_{i=1}^N C_{P_{NET_i}} A_i [(y_i - y_{CG}) \cos(\theta_i) + (z_i - z_{CG}) \sin(\theta_i)] \\ - \sum_{j=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s'_j [(y_{LE} - y_{CG}) \Omega_{L_j} + (z_{LE} - z_{CG}) \Omega_{Y_j}] \\ - C_T^2 \sum_{k=1}^{NC_T} C_{s_k} (\Delta x/C_T)_k [(y_T - y_{CG}) T_{L_k} + (z_T - z_{CG}) T_{Y_k}]$$

Pitching Moment:

$$C_{m \bar{c} S_{REF}} = -F_1 \sum_{i=1}^N C_{P_{NET_i}} A_i (x_i - x_{CG}) \cos(\theta_i) \\ - \sum_{j=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s'_j (x_{LE} - x_{CG}) \Omega_{L_j} - C_T^2 \sum_{k=1}^{NC_T} C_{s_k} \Delta(x/C_T)_k T_{Y_k}$$

Yawing Moment:

$$C_{n b S_{REF}} = F_2 \sum_{i=1}^N C_{P_{NET_i}} A_i (x_i - x_{CG}) \sin(\theta_i) \\ + \sum_{j=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s'_j (x_{LE} - x_{CG}) \Omega_{Y_j} + C_T^2 (x_T - x_{CG})_{c.p.} \sum_{k=1}^{NC_T} C_{s_k} \Delta(x/C_T)_k T_{Y_k}$$

where N is the number of vortex panels on half of a symmetrical component (or total for an asymmetrical component) and F_1 , F_2 are given by

symmetric loading:	F_1	= 1	asymmetric geometry
		= 2	symmetric geometry
	F_2	= 1	asymmetric geometry
		= 0	symmetric geometry
antisymmetric loading:	F_1	= 1	asymmetric geometry
		= 0	symmetric geometry
	F_2	= 1	asymmetric geometry
		= 2	symmetric geometry

For the leading and side edge vortex terms, N_s is the total number of spanwise panels for both component halves, NC_T is the number of tip chordwise panels, $x_{T_{c.p.}}$ is the axial location of tip vortex center of

pressure, $\Delta s' = \Delta s \sqrt{1 + T^2}$ and the rotation factors Ω and T are derived in appendix B and defined below.

Leading Edge Vortex Rotation:

$$\begin{aligned}\Omega_L &= -\sin \alpha (\cos \Lambda \cos \delta) + \cos \alpha (\cos \theta \sin \delta - \sin \theta \sin \Lambda \cos \delta) \\ &+ A_o/|A_o| [\sin \alpha (\cos \Lambda \sin \delta) + \cos \alpha (\cos \theta \cos \delta + \sin \theta \sin \Lambda \sin \delta)] \\ \Omega_Y &= \cos \theta \sin \Lambda \cos \delta + \sin \theta \sin \delta \\ &+ A_o/|A_o| [\sin \theta \cos \delta - \cos \theta \sin \Lambda \sin \delta]\end{aligned}$$

where δ is the slope angle of the leading edge camber line and the sign of coefficient A_o (from equation 13) is used to determine the direction of vortex rotation.

Side Edge Vortex Rotation:

$$\begin{aligned}T_L &= \pm \cos \alpha \sin \theta + C_{n_0}/|C_{n_0}| (\sin \alpha \sin \delta + \cos \alpha \cos \theta \cos \delta) \\ T_Y &= \pm \cos \theta + C_{n_0}/|C_{n_0}| \sin \theta \cos \delta\end{aligned}$$

where δ is the slope angle of the tip camber line, \pm is plus for the left side and negative for the right side of the configuration and the sign of coefficient C_{n_0} (from equation 14) is used to determine the direction of vortex rotation.

The x-coordinate of the center of pressure is given by

$$x_{c.p.} = -C_m \bar{c} / C_L + x_{CG}$$

For interference shell components, the total forces and moments of the corresponding isolated body are added to those of the shell.

Jet reaction forces and moments are obtained from a spanwise summation of the jet flap section characteristics:

Lift:

$$C_{L_{JET}} = \frac{F_1}{S_{REF}} \sum_{i=1}^N (C_{\mu c})_i \delta_{jT_i} \Delta s_i \cos \theta_i$$

Side Force:

$$C_{Y_{JET}} = \frac{-F_2}{S_{REF}} \sum_{i=1}^N (C_{\mu c})_i \delta_{jT_i} \Delta s_i \sin \theta_i$$

Rolling moment:

$$C_{\ell_{JET}} = \frac{-F_2}{b S_{REF}} \sum_{i=1}^N (C_{\mu c})_i \delta_{jT_i} \Delta s_i [\cos \theta_i (y_i - y_{CG}) + \sin \theta_i (z_i - z_{CG})]$$

Pitching moment:

$$C_{m_{JET}} = \frac{-F_1}{\bar{c} S_{REF}} \sum_{i=1}^N (C_{\mu c})_i \delta_{jT_i} \Delta s_i \cos \theta_i (x_i - x_{CG})$$

Yawing moment:

$$C_{n_{JET}} = \frac{F_2}{b S_{REF}} \sum_{i=1}^N (C_{\mu c})_i \delta_{jT_i} \Delta s_i \sin \theta_i (x_i - x_{CG})$$

where N is the number of spanwise jet flap stations and F_1 and F_2 are as previously defined.

The forces and moments for the complete configuration are obtained by summing those of the individual components.

DRAG ANALYSIS

Estimation of configuration aerodynamic efficiency requires the calculation of drag. The analysis separates the computation into skin friction and pressure drag components that are assumed to be independent of each other. The following form is considered and produces nonparabolic polars as a result of the incorporation of edge force considerations.

$$C_D = C_{D_{\text{viscous}}} + C_{D_{\text{wave}}} + C_{D_{\text{base}}} + C_{D_{\text{lift}}}$$

The specific techniques used for the various drag evaluations are discussed below.

SKIN FRICTION

Several well established semiempirical techniques for the evaluation of adiabatic laminar and turbulent flat plate skin friction at incompressible and compressible speeds are used to estimate the viscous drag of advanced aircraft using a component buildup approach. A specified transition point calculation option is provided in conjunction with a matching of the momentum thickness to link the two boundary layer states. For the turbulent condition, the increase in drag due to distributed surface roughness is treated using uniformly distributed sand grain results. Component thickness effects are approximated using experimental data correlations for two-dimensional airfoil sections and bodies of revolution.

Considerations such as separation, component interference, and discrete protuberances (e.g. antennas, drains, aft facing steps, etc.) must be accounted for separately.

In the following, a discussion is presented for a single component evaluation in order to simplify writing of the equations and eliminate multiple subscripting. The total result is obtained by a surface area weighted summation of the various component analyses as described on page 44.

Laminar/Transition

A specified transition option is provided in the program. The principal function of the calculation is to provide the conditions required to initialize the turbulent solution. In particular, the transition point length and momentum thickness Reynolds numbers are required.

$$R_{x_{\text{TRAN}}} = R(x_{\text{TRAN}}/L)L$$

$$R_{\theta_{\text{TRAN}}} = 0.664 \sqrt{R_{x_{\text{TRAN}}} C^*}$$

where

$$C^* = (\mu^*/\mu_\infty)(T_\infty/T^*)$$

$$T^*/T_\infty = 1 + 0.72 [(T_r/T_\infty) - 1]$$

$$T_r/T_\infty = 1 + \sqrt{\text{Pr}} (\gamma-1)/2 M_\infty^2 = 1 + 0.851(\gamma-1)/2 M_\infty^2$$

$$\mu = 2.270 \times 10^{-8} T^{3/2}/(T+198.6) \text{ lb-sec/ft}^2$$

This solution is based on the laminar Blasius result (8, chapter VII) in conjunction with Eckert's compressibility transformation⁹. This option permits an assessment of the reduction in skin friction drag if laminar flow can be maintained for the specified extent. It does not establish the likelihood that such a condition will be realized in practice or to what extent.

Turbulent

Smooth and distributed rough surface options have been provided in the analysis. In either case, the solution is initialized by matching the momentum thickness at the transition point produced by the laminar solution. That is, an effective origin (commonly referred to as a virtual origin) is established for the turbulent analysis.

For the hydraulically smooth case

$$C_F R_{\Delta x} = 2 R_{\theta_{\text{TRAN}}}$$

$$R_{\Delta x} = C_F R_{\Delta x} / C_F \quad \begin{array}{l} C_F \text{ from equation (19)} \\ \text{for known } C_F R_{\Delta x} \end{array}$$

$$\Delta x = R_{\Delta x} / R$$

$$\ell = L - X_{\text{TRAN}} + \Delta x$$

$$R_\ell = (R)(\ell)$$

solve for C'_F using,

$$0.242[\sin^{-1}\alpha + \sin^{-1}\beta] / [(\gamma-1)/2M_\infty^2 C'_F]^{1/2} = \log_{10}(C'_F R_\ell) - \omega \log_{10}(T_r/T_\infty) \quad (19)$$

then,

$$C_F = 2\theta_{x \rightarrow \infty}/L = (2\theta_{\text{TE}}/\ell)(\ell/L) = C'_F(\ell/L)$$

where

$$x_{\text{TRAN}} = R^{-1} R_{x_{\text{TRAN}}}$$

$$\alpha = (2A^2 - B)[B^2 + 4A^2]^{-1/2}$$

$$\beta = B[B^2 + 4A^2]^{-1/2}$$

$$A^2 = (\gamma-1)/2 M_\infty^2 (T_\infty/T_r)$$

$$B^2 = [1 + (\gamma-1)/2 M_\infty^2](T_\infty/T_r) - 1$$

$$r = 0.88$$

$$\omega = 0.76$$

The compressible turbulent flat plate method used here is that proposed by Van Driest¹⁰ based on the Von Karman mixing length hypothesis in conjunction with the Squire-Young formulation for profile drag (8, chapter XXIV) as applied to a flat plate.

For the distributed rough case

$$\Delta \tilde{x} = x_{\text{TRAN}}$$

$$C_{F_r} = [1.89 + 1.62 \log_{10}(\Delta \tilde{x}/K_s)]^{-2.5} [1 + r(\gamma-1)/2 M_\infty^2]^{-1}$$

$$R_{\Delta x} = 2 C_{F_r}^{-1} R_{\theta_{\text{TRAN}}}$$

$$\Delta x = R^{-1} R_{\Delta x_{i+1}}$$

$$\ell = L - x_{\text{TRAN}} + \Delta x$$

$$C'_F = [1.89 + 1.62 \log_{10}(\ell/K_s)]^{-2.5} [1 + r(\gamma-1)/2 M_\infty^2]^{-1}$$

$$C_F = C'_F (\ell/L)$$

$$C_F = \text{MAX} [C_{F_{\text{SMOOTH}}}, C_{F_{\text{ROUGH}}}]$$

The turbulent flat plate method used here is that of Schlichting (8, chapter XXI) which is based on a transposition of Nikuradse's densely packed sand grain roughened pipe data. The effect of compressibility is due to the reduction in density at the wall as proposed by Goddard.¹¹ The selection of the equivalent sand grain roughness for a given manufacturing surface finish is made with the aid of Table II which was taken from Clutter.¹²

TABLE II

<u>Type of Surface</u>	<u>Equivalent Sand Roughness</u> <u>K_s (inches)</u>
Aerodynamically smooth	0
Polished metal	0.02 - 0.08 x 10 ⁻³
Natural sheet metal	0.16 x 10 ⁻³
Smooth matte paint, carefully applied	0.25 x 10 ⁻³
Standard camouflage paint, average application	0.40 x 10 ⁻³
Camouflage paint, mass-production spray	1.20 x 10 ⁻³
Dip-galvanized metal surface	6 x 10 ⁻³
Natural surface of cast iron	10 x 10 ⁻³

Thickness Corrections

The foregoing evaluations produce an estimate of the shearing forces on a flat plate (at zero angle of attack) for a variety of conditions. As an actual aircraft has a finite thickness, an estimate of pressure gradient effects on skin friction and boundary layer displacement pressure drag losses is required. A common procedure for accomplishing this and the one which will be used here is based on non-lifting experimental correlations for symmetric two-dimensional airfoils and axisymmetric bodies. The following relations derived by Horner (13, chapter VI) are used, respectively.

$$K = C_d / (2C_F) = 1 + K_1 (t/c) + 60 (t/c)^4$$

$$= C_D / C_{D_F} = 1 + 1.5 (d/L)^{3/2} + 7 (d/L)^3$$

Horner recommends $K_1 = 2$ for airfoils with maximum thickness at 30% chord and $K_1 = 1.2$ for NACA 64 and 65 series airfoils. In this regard, the best information available to an analyst for his particular contour should be used. This is especially true for modern high performance shapes such as the supercritical airfoil.

Total Viscous Drag

The aircraft total viscous-drag coefficient is estimated by a sum of the preceding analysis over all components (i.e. wing, fuselage, vertical tail, etc.). That is

$$C_{D_{\text{viscous}}} = \sum_{j=1}^N C_{F_j} (S_j/S_{\text{REF}})^{K_j}$$

The component length used in the calculation of the skin friction coefficient is the local chord for planar component segments and the physical length for bodies and nacelles.

BASE DRAG

Blunt base increments are estimated at subsonic and supersonic speeds by

$$\Delta C_{D_{\text{BASE}}} = -C_{P_{\text{BASE}}} S_{\text{BASE}} / S_{\text{REF}}$$

where

$$-C_{P_{\text{BASE}}} = 0.139 + 0.419 (M_{\infty} - 0.161)^2; \quad M_{\infty} < 1$$

$$-C_{P_{\text{BASE}}} = M_{\infty}^{-2} - 0.57 M_{\infty}^{-4}; \quad M_{\infty} \geq 1$$

The expressions for the base pressure coefficient are derived from correlation of flight test results for the X-15, various lifting bodies, and the space shuttle. Power effects are treated as reductions in base area in the present analysis.

POTENTIAL DRAG

One hundred percent suction drag due to lift and supersonic wave drag due to thickness can be evaluated by integration of the momentum flux through a large circular cylinder centered on the x-axis and whose radius approaches infinity (figure 9).

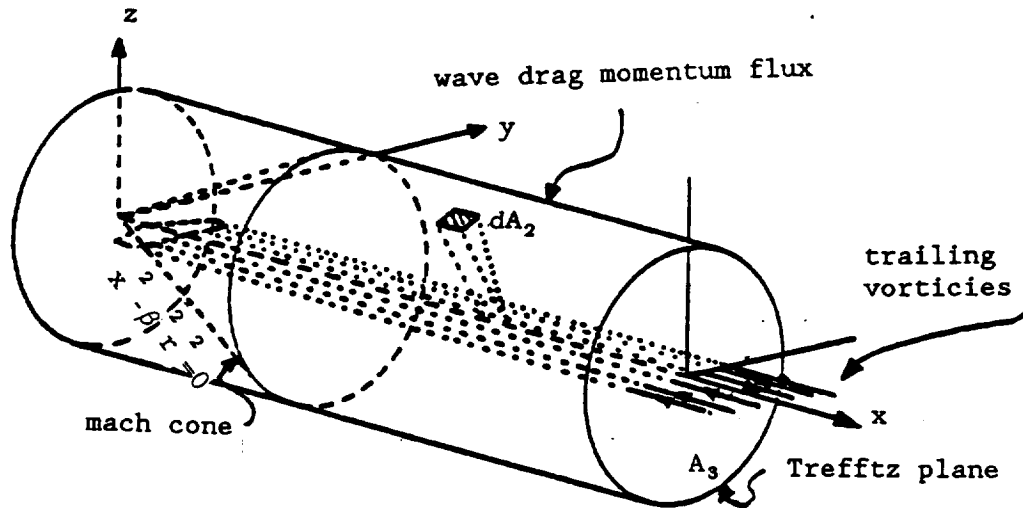


Figure 9. Integration of Momentum Flux Through Large Circular Cylinder.

The resulting expression for the total pressure drag is

$$C_{D_{REF}} S_{REF} = -2 \iint_{A_2} \Phi_x \Phi_r dA_2 + \iint_{A_3} (\Phi_y^2 + \Phi_z^2) dA_3 = C_{D_W} S_{REF} + C_{D_V} S_{REF}$$

The first term represents the wave drag due to momentum losses thru the side of the cylinder caused by standing pressure waves at supersonic speeds. The second term represents the vortex drag which arises from the kinetic energy left behind in the Trefftz plane by the system of trailing vortices.

Vortex Drag

The vortex drag may be computed when the distribution of trailing vorticity in the Trefftz plane is known. The assumptions of linearized thin wing theory result in a vortex sheet which extends directly downstream of all lifting surfaces. By changing a surface integral for kinetic energy to a line integral over the vortex sheet in the Trefftz plane the following expressions for lift and drag result.

$$C_L = (c_{AVG}/S_{REF}) \int_C \bar{C}_n(\eta) \cos \theta(\eta) d\eta$$

$$C_{DV} = (c_{AVG}/S_{REF}) \int_C \bar{C}_n(\eta) w_\infty(\eta) d\eta$$

where

C vortex sheet contour

\bar{C}_n weighted section normal force coefficient $C_n(c/c_{AVG})$

w_∞ asymptotic normal velocity on the vortex sheet

η vortex sheet branch coordinate

θ inclination of vortex sheet with respect to y-axis

The analysis computes the normal velocity on the vortex sheet, $w_{\infty i}$ by assuming the vortex sheet is composed of finite trailing horseshoe vortices whose strength is proportional to the local section $c_n(s)$. The normal velocity is computed at a control point located midway between the trailing vortex segments (figure 10).

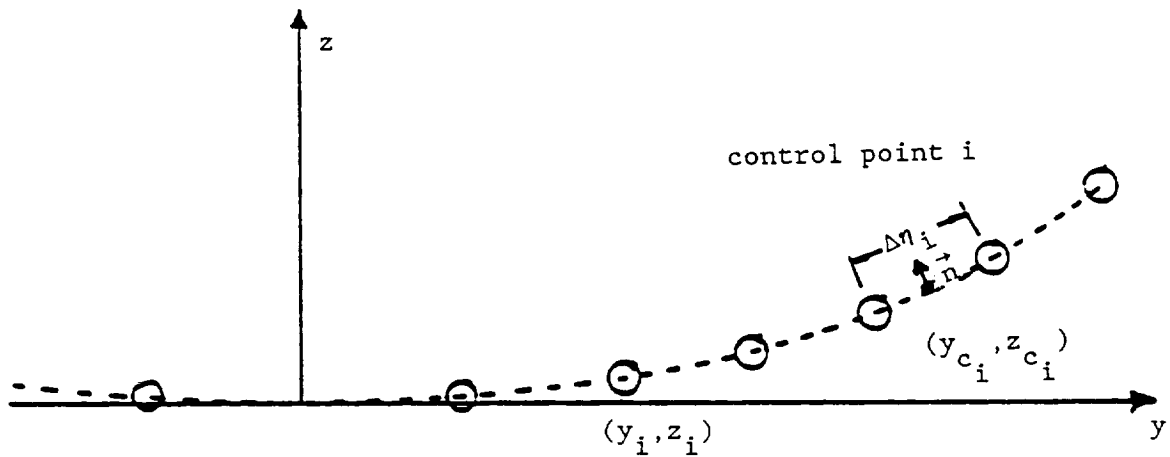
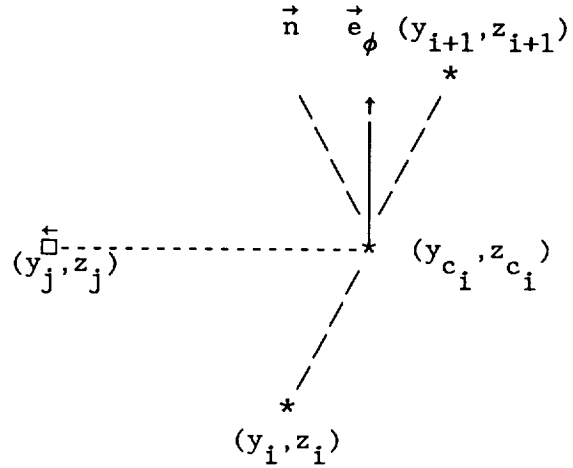


Figure 10. Trefftz Plane Vortex Wake Nomenclature.

$$\vec{V} = c_{AVG} \Gamma_j / (2\pi r) \vec{e}_\phi$$

$$\Gamma = [\bar{c}_{n_j} - \bar{c}_{n_i}] / 2$$



$$\vec{r} = [(y_{c_i} - y_j)\vec{e}_y + (z_{c_i} - z_j)\vec{e}_z]$$

$$\Delta \vec{s}_i = (y_{i+1} - y_i)\vec{e}_y + (z_{i+1} - z_i)\vec{e}_z$$

$$= \Delta y_i \vec{e}_y + \Delta z_i \vec{e}_z$$

$$\vec{e}_\phi = [-(z_{c_i} - z_j)\vec{e}_y + (y_{c_i} - y_j)\vec{e}_z] / r$$

$$\vec{n} = \Delta s_i^{-1} [-\Delta z_i \vec{e}_y + \Delta y_i \vec{e}_z]$$

$$w_{\infty i} = \vec{V} \cdot \vec{n} = c_{AVG} / (2\pi r^2 \Delta s_i) \left\{ (z_{c_i} - z_j) \Delta z_i + (y_{c_i} - y_j) \Delta y_i \right\} \Gamma_j = A_{ij} \Gamma_j$$

$$\text{Therefore} \quad C_D = \sum_i w_{\infty i} \bar{c}_{n_i} \Delta \eta_i$$

$$\text{where} \quad w_{\infty i} = \sum_j A_{ij} \Gamma_j$$

Wave Drag

The integral for wave drag

$$C_{D_W} S_{REF} = -2 \iint_{A_2} \phi_x \phi_r r dx d\theta$$

may be simplified by allowing the cylindrical surface of integration to recede infinitely far from the disturbance. Under these conditions, the spatial singularity simulations can be reduced to a series of one-dimensional distributions. The basis for this reduction is the finding by Hayes (14) that the potential and the gradients of interest induced by a singularity along an arbitrary trace on a distant control surface, say PP' of figure 11 (or alternately described by the cylindrical angle θ), is invariant to a finite translation along the surface of a hyperboloid emanating from the trace and passing through the singularity. As the apex of the hyperboloid is a great distance away, the aforementioned movement is along a surface which is essentially plane; it will be henceforth referred to as an "oblique plane". Since a singularity is a solution of a linear differential equation, all singular solutions which lie on the surface of

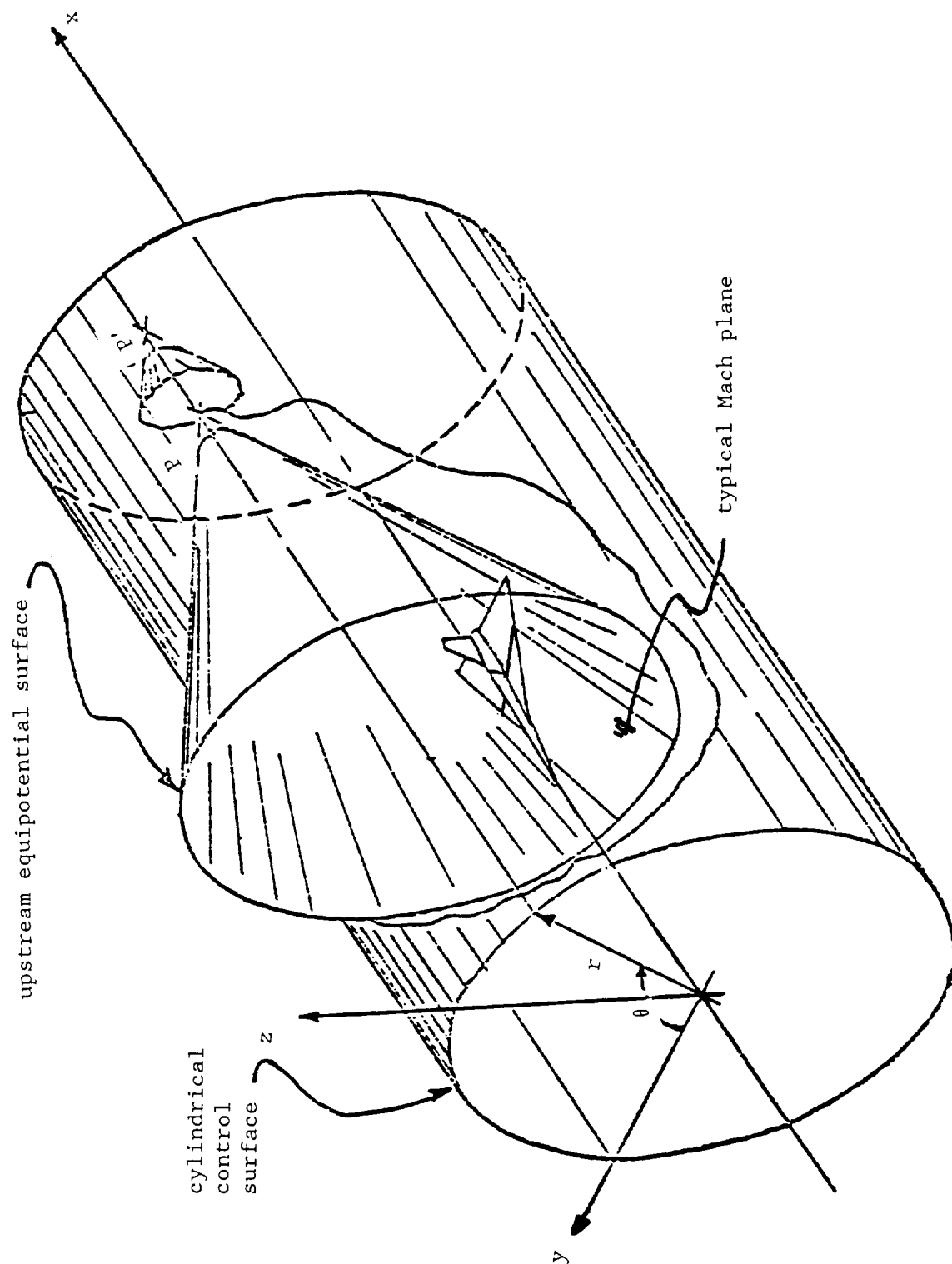


Figure 11. Distant Control Surface Geometry.

the same hyperboloid (oblique plane) may thus be grouped to form a single equivalent point singularity whose strength is equal to the algebraic sum of the individual strengths and which induces the same potential (momentum) along the trace as the group of individual singularities.

This finding provides the basic technique for reducing a general spatial distribution of singularities to a series of equivalent lineal distributions. This is accomplished by surveying the three-dimensional distribution longitudinally at a series of fixed cylindrical angles, θ . At each angle, the survey produces an equivalent lineal distribution by systematically cutting the spatial distribution at a series of longitudinal stations along its length. At each cut, the group of intercepted singularities is collapsed along the "oblique plane" to form one of the equivalent point singularities comprising the lineal distribution.

The far-field expression for the wave drag of a general system of lift, and side force elements is

$$C_{D_W} S_{REF} = (4\pi U)^{-2} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h'_e(\epsilon_1, \theta) h'_e(\epsilon_2, \theta) \ln|\epsilon_1 - \epsilon_2| d\epsilon_1 d\epsilon_2 d\theta$$

where

$$h_e(\epsilon, \theta) = f(\epsilon, \theta) - g_z(\epsilon, \theta) \sin \theta - g_y(\epsilon, \theta) \cos \theta$$

is the equivalent lineal singularity strength at the cylindrical angle θ

$$f(\epsilon, \theta) = \text{equivalent source strength per unit length}$$

$$\beta^{-1} U g_z(\epsilon, \theta) = \text{equivalent lifting element strength per unit length}$$

$$\beta^{-1} U g_y(\epsilon, \theta) = \text{equivalent side force strength per unit length}$$

These strengths are deduced from the three dimensional singularity distributions by application of the superposition principle along equipotential surfaces. For a distant observer such surfaces are planar in the vicinity of the singularity configuration. The individual singularity strengths are related to the object under consideration by the requirement of flow tangency at the solid boundary. Lomax (15) derived the following approximate expressions between the equivalent singularity strengths and a slender lifting object.

$$f(\epsilon, \theta) = U \partial / \partial \epsilon [A(\epsilon, \theta)]$$

$$g_z(\epsilon, \theta) = (\beta U) / 2 \int_c C_p dy$$

$$g_y(\epsilon, \theta) = (\beta U) / 2 \int_c C_p dz$$

where (see figure 12)

$A(\epsilon, \theta)$ - is the Y-Z projection of the obliquely cut cross-sectional area

c - is the contour around the surface in the oblique cut

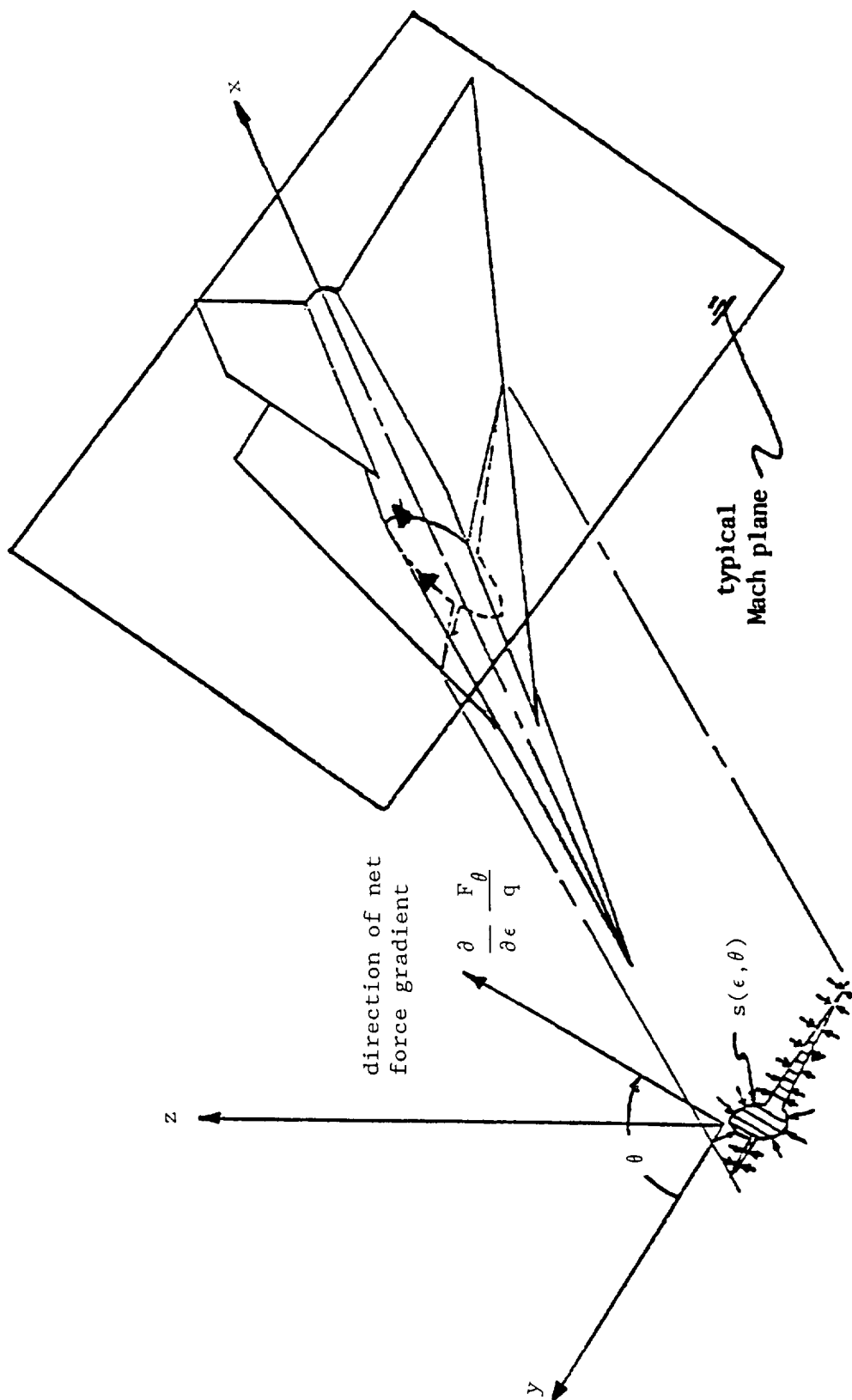


Figure 12. Areas and Forces Pertinent to the Evaluation of Wave Drag from the Far Field Point of View.

Utilizing the singularity strength expressions derived by Lomax, the following expression for wave resistance based on the far field theory of Hayes is obtained

$$C_{D_W} S_{REF} = \frac{1}{(4\pi)^2} \int_0^{2\pi} \int_{-L(\theta)}^{L(\theta)} \int_{-L(\theta)}^{L(\theta)} \left\{ \frac{\partial^2}{\partial \epsilon_1^2} [A(\epsilon_1, \theta)] \right. \\ \left. - \beta/2 \frac{\partial}{\partial \epsilon_1} [\sin \theta \int_C C_p(\epsilon_1, \theta) dy + \cos \theta \int_C C_p(\epsilon_1, \theta) dz] \right\} \\ \left\{ \frac{\partial^2}{\partial \epsilon_2^2} [A(\epsilon_2, \theta)] - \beta/2 \frac{\partial}{\partial \epsilon_2} [\sin \theta \int_C C_p(\epsilon_2, \theta) dy + \cos \theta \int_C C_p(\epsilon_2, \theta) dz] \right\} \\ \ln |\epsilon_1 - \epsilon_2| d\epsilon_1 d\epsilon_2 d\theta$$

In order to facilitate subsequent discussion, the above result is manipulated into the following form

$$C_{D_W} S_{REF} = [4\pi^2 L^2(\theta)]^1 \int_0^{2\pi} \int_0^1 \int_0^1 \frac{\partial^2}{\partial \epsilon_1^2} [A_e(\epsilon_1, \theta)] \frac{\partial^2}{\partial \epsilon_2^2} [A_e(\epsilon_2, \theta)] \ln |\epsilon_1 - \epsilon_2| d\epsilon_1 d\epsilon_2 d\theta \quad (20)$$

where the effective A_e is given by,

$$A_e(\epsilon, \theta) = A(\epsilon, \theta) - \beta/2 \int_0^\epsilon \int_C C_p(\epsilon, \theta) [\sin \theta dy + \cos \theta dz] d\epsilon$$

A requirement for this transformation is that

$$A'_e(0, \theta) = A'_e(L, \theta) = 0$$

In accordance with equation 20, the wave drag of a configuration is the average of the wave drag of a series of equivalent bodies of revolution. The drag of each of these bodies is calculated from a knowledge of its longitudinal distribution of normal cross-sectional area. For each equivalent body, these areas are defined to be the frontal projection of the areas and the accumulation of pressure force in the theta direction intercepted on the original configuration by a system of parallel oblique planes each inclined at the given Mach angle. The common polar angle (θ) of the system identifies the equivalent body under consideration.

Nacelles are assumed to swallow air supersonically. That is, the duct is operating at a mass flow ratio of unity. Consistent with this assumption, the equivalent body cross-sectional area distribution is increased by the oblique projected duct capture area at all stations ahead of the duct which are intercepted by an oblique plane.

Blunt base components are extended (maintaining constant cross-sectional area) sufficiently far downstream to prevent flow closure around the base.

In addition to a geometric description, a definition of the pressure distribution acting on the configuration is required. The vortex panel analysis is used for this purpose. The thickness pressures for planar

components have tacitly been neglected under the assumption that the surfaces are sufficiently thin that the net pressure coefficient is representative of pressure acting on the oblique section.

Estimation of the wave drag based on equation 20 depends on solution of integrals of the type

$$I = \frac{1}{2\pi} \int_0^1 \int_0^1 G''(x_1) G''(x_2) \ln|x_1 - x_2| dx_1 dx_2$$

of a numerically given function $G(x)$. Evaluation of such forms has been studied by Eminton^{16, 17} for functions having $G'(x)$ continuous on the interval $(0,1)$ and $G'(0) = G'(1) = 0$. In such situations, $G'(x)$ can be expanded in a Fourier sine series. It can then be shown that

$$I = \sum_{N=1}^{\infty} N A_N^2$$

where

$$A_N = \frac{2}{\pi} \int_0^{\pi} G'(x) \sin(N\psi) d\psi$$

Eminton then solved for the value of the Fourier coefficients which result in I being a minimum, subject to the condition that the resulting series for $G(x)$ be exact for an arbitrarily specified set of points $(0,1)$, ϵ_i , $i=1,n$.

This approach produces the following result

$$I = \frac{4}{\pi} [G(1) - G(0)]^2 + \pi \sum_{i=1}^n \sum_{j=1}^n C_i C_j f_{ij}$$

where

$$C_i = G(\epsilon_i) - G(0) - [G(1) - G(0)] \mu_i$$

$$\mu_i = \frac{1}{\pi} \left\{ \cos^{-1}(1-2\epsilon_i) - \frac{2(1-2\epsilon_i)}{\sqrt{\epsilon_i(1-\epsilon_i)}} \right\}$$

$$\epsilon_i = \frac{i}{(n+1)} ; \quad 1 \leq i \leq n$$

$$f_{ij} = [P_{ij}]^{-1}$$

$$P_{ij} = -(\epsilon_i - \epsilon_j)^2 \frac{1}{2} \ln \left\{ \frac{[\epsilon_i + \epsilon_j - 2\epsilon_i \epsilon_j + 2\sqrt{\epsilon_i \epsilon_j (1-\epsilon_i)(1-\epsilon_j)}]}{[\epsilon_i + \epsilon_j - 2\epsilon_i \epsilon_j - 2\sqrt{\epsilon_i \epsilon_j (1-\epsilon_i)(1-\epsilon_j)}]} \right\} \\ + 2(\epsilon_i + \epsilon_j - \epsilon_i \epsilon_j) \sqrt{\epsilon_i \epsilon_j (1-\epsilon_i)(1-\epsilon_j)}$$

The solution of equation 20 for wave drag is accomplished by use of the following identities.

$$G(\epsilon_i, \theta) = A_e(\epsilon_i, \theta)$$

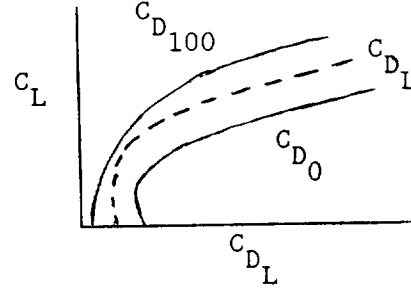
$$C_{D_W}(\theta) S_{REF} = I(\theta) / L^2(\theta)$$

$$C_{D_W} = \frac{1}{2\pi} \int_0^{2\pi} C_{D_W}(\theta) d\theta = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} C_{D_W}(\theta) d\theta$$

DRAG DUE TO LIFT

In the discussion which follows, certain limiting (zero and one hundred percent suction), and attainable edge force polars are defined. They are related to one another as indicated in the following sketch.

If $C_{D100} > C_{D0}$, there is a slenderness problem in the C_{D100} calculation. In this case the C_{D0} calculation is considered more reliable and the C_{DL} curve is found relative to C_{D0} and the suction level.



The fixed one hundred percent suction drag due to lift (i.e. C_{D100}) is given by equations 20 through 22. Specifically

$$C_{D100} = C_{DV} \quad M < 1$$

$$= C_{DV} + C_{DW} - C_{DW}|_{\text{THICKNESS}} \quad M > 1$$

The zero suction drag due to lift is calculated by numerically integrating the net pressure distribution times the projected area in the streamwise direction over each of the planar surfaces. The following block integration scheme is used to sum over all quadrilateral panels.

$$C_{D0} = F_1 (S_{REF})^{-1} \sum_{i=1}^N C_{Pi} A_i \alpha_i$$

where

$$C_{Pi} = C_{Pi_{\alpha=0}} + \alpha \frac{\partial C_{Pi}}{\partial \alpha} + \delta \frac{\partial C_{Pi}}{\partial \delta}$$

and

$$\alpha_i = \alpha_{0i} + \alpha + \sigma_i \delta$$

α_{0i} is due to twist and camber, δ is the control surface deflection, and $\sigma_i = 1$ for control surface panels and $\sigma_i = 0$ for non control surface panels. $F_1 = 2$ for symmetric geometries and $F_1 = 1$ for asymmetric geometries.

Edge forces are neglected in this evaluation.

The drag due to lift for the total configuration is based on linearized potential (100 percent leading edge suction) calculations plus corrections to account for suction losses and associated edge vortex forces

$$C_{D_L} = C_{D_{100}} + S_{REF}^{-1} \sum_{i=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s_j' \Omega_{D_j} + C_T^2 (S_{REF})^{-1} \sum_{k=1}^{Nc_T} C_{s_k} \Delta(x/c_T)_k T_{D_k}$$

$$= C_{D_0} + S_{REF}^{-1} \sum_{i=1}^{Ns} (1-K_{s_j}) C_{s_j} c_j \Delta s_j' \Omega_{D_j} + C_T^2 (S_{REF})^{-1} \sum_{k=1}^{Nc_T} C_{s_k} \Delta(x/c_T)_k T_{D_k}$$

and the leading edge and side edge rotation factors, Ω_D and T_D , are (see derivation in Appendix B)

$$\Omega_D = \cos \alpha \cos \Lambda \cos \delta + \sin \alpha (\cos \theta \sin \delta - \sin \theta \sin \Lambda \cos \delta)$$

$$+ A_0/|A_0| [-\cos \alpha \cos \Lambda \sin \delta + \sin \alpha (\cos \theta \cos \delta + \sin \theta \sin \Lambda \sin \delta)]$$

where δ is the slope angle of the camber line perpendicular to the leading edge and the sign of coefficient A_0 from equation 13 is used to determined the direction of vortex rotation.

$$T_D = \pm \sin \alpha \sin \theta - C_{n_0} |C_{n_0}|^{-1} (-\cos \alpha \sin \delta + \sin \alpha \cos \theta \cos \delta)$$

where δ is the chordwise slope angle of the tip camber line, plus refers to the left side and negative to the right side of the configuration and the sign of coefficient C_{n_0} from equation 14 is used to determine the direction of vortex rotation.

An estimate of the average level of leading edge suction for the complete configuration is based on the following equation:

$$\text{SUCTION} = (\bar{C}_{D_L} - C_{D_L}) / (\bar{C}_{D_L} - C_{D_{100}}), \text{ where } \bar{C}_{D_L} = C_{D_L} \text{ for } K_s = 0$$

$$= 1.0 ; \Rightarrow \text{L.E. Suction, if any, is totally recovered.}$$

$$< 1.0 ; \Rightarrow \text{L.E. Suction is partly recovered, the remainder is converted to vortex lift and drag.}$$

HYPERSONIC

High Mach number analysis is based on non-interfering constant pressure¹⁸ finite element analysis.

An arbitrary configuration is approximated by a system of plane quadrilateral panels as indicated in figure 13.

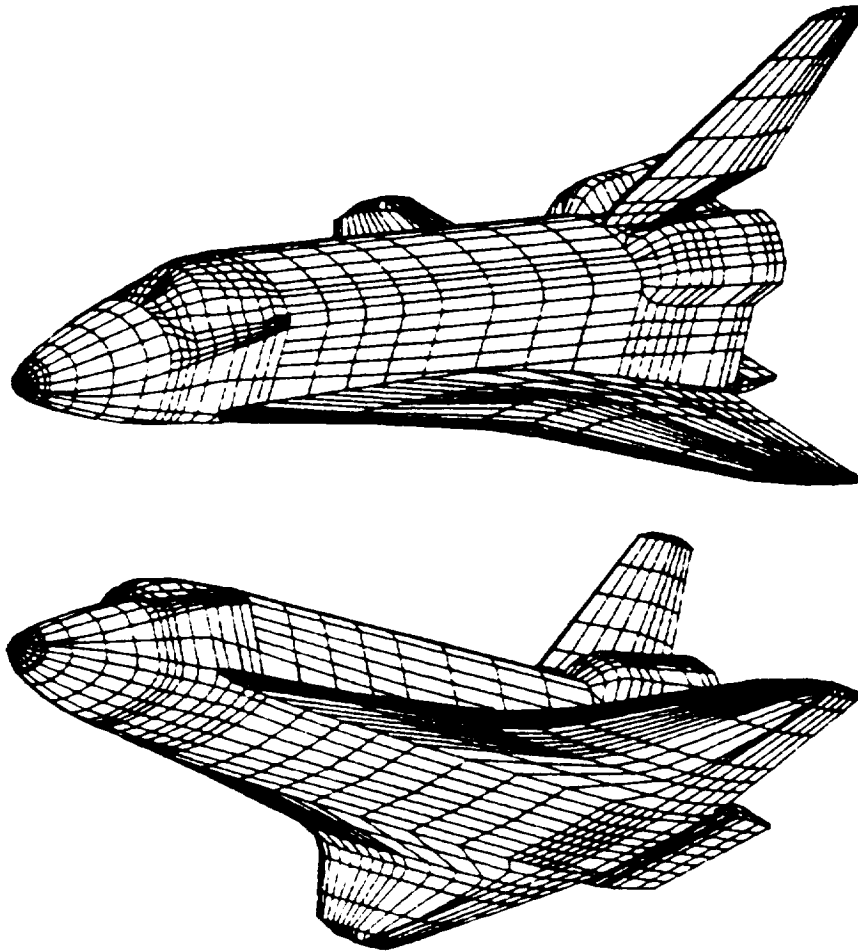


Figure 13. Configuration Represented by Surface Quadrilateral Panels.

The pressure acting on each panel of a vehicle component is evaluated by a specified compression-expansion method selected from the following options.

Impact Flow

1. Modified Newtonian
2. Modified Newtonian+Prandtl-Meyer
3. Tangent wedge
4. Tangent-wedge empirical
5. Tangent-cone empirical
6. OSU blunt body empirical
7. Van Dyke Unified
8. Blunt-body shear force
9. Shock-expansion
10. Free molecular flow
11. Input pressure coefficient
12. Hankey flat-surface empirical
13. Delta wing empirical
14. Dahlem-Buck empirical
15. Blast wave
16. Modified tangent-cone

Shadow Flow

1. Newtonian ($C_p = 0$)
2. Modified Newtonian+Prandtl-Meyer
3. Prandtl-Meyer from free-stream
4. OSU blunt body empirical
5. Van Dyke Unified
6. High Mach base pressure
7. Shock-expansion
8. Input pressure coefficient
9. Free molecular flow

A discussion of the various methods is presented in appendix C. Specific analysis recommendations are provided by the program on a component by component basis.

In each method, the only geometric parameter required for determining panel pressure is the impact angle, δ , that the quadrilateral makes with the free-stream flow or the change in angle of a panel from a previous point where

$$\delta = \pi/2 - \theta$$

$$\cos \theta = (\vec{n} \cdot \vec{V}) / (|\vec{n}| |\vec{V}|)$$

and

$$\vec{n} = n_x \vec{i} + n_y \vec{j} + n_z \vec{k}$$

$$\vec{V} = \vec{V}_\infty - \vec{\Omega} \times \vec{r}$$

$$\vec{V}_\infty = (V_\infty \cos \alpha \cos \beta) \vec{i} - (V_\infty \sin \beta) \vec{j} + (V_\infty \sin \alpha \cos \beta) \vec{k}$$

$$\vec{\Omega} = p \vec{i} - q \vec{j} - r \vec{k}$$

$$\vec{r} = (x - x_{CG}) \vec{i} + (y - y_{CG}) \vec{j} + (z - z_{CG}) \vec{k}$$

Panel switching between impact or shadow conditions is based on $\delta > 0$ in the former case and $\delta < 0$ in the latter.

AERODYNAMIC CHARACTERISTICS

The pressure on each panel is calculated independent of all other panels (except the shock-expansion method). If the vehicle is rotating, the local pressure coefficient must be corrected to free-stream conditions. That is

$$C_P = C_{P_{\text{local}}} |\vec{V}|^2 / V_\infty^2$$

Vehicle component forces, which are in the body axis system, are obtained by summing panel forces

$$\Delta C_x = S_{\text{REF}}^{-1} \sum C_P n_x A$$

$$\Delta C_y = S_{\text{REF}}^{-1} \sum C_P n_y A$$

$$\Delta C_z = S_{\text{REF}}^{-1} \sum C_P n_z A$$

$$\Delta C_\ell = (b S_{\text{REF}})^{-1} \left\{ \sum C_P (z - z_{\text{CG}}) n_y A + \sum C_P (y - y_{\text{CG}}) n_z A \right\}$$

$$\Delta C_m = -(c S_{\text{REF}})^{-1} \left\{ \sum C_P (x - x_{\text{CG}}) n_z A + \sum C_P (z - z_{\text{CG}}) n_x A \right\}$$

$$\Delta C_n = (b S_{\text{REF}})^{-1} \left\{ \sum C_P (x - x_{\text{CG}}) n_y A - \sum C_P (y - y_{\text{CG}}) n_x A \right\}$$

where

A = panel area

x, y, z = coordinates of panel centroid

Configuration buildup and total vehicle coefficients are obtained by appropriate summation of component contributions.

The conversion from the body axis system to the wind axis system for the lift and drag coefficients is based on the standard trigonometric relations.

$$C_D = C_x \cos \alpha \cos \beta - C_y \sin \beta + C_z \sin \alpha \cos \beta$$

$$C_L = -C_x \sin \alpha + C_z \cos \alpha$$

The vehicle static stability derivatives, which are in the body axis system, are calculated by the method of small perturbations. Since the basic force and moment characteristics are non-linear, these parameters vary with attitude angle

$$C_{x_{\alpha}} = [(C_x)_{\alpha+\Delta\alpha} - (C_x)_{\alpha}] / \Delta\alpha \quad (\text{Axial})$$

$$C_{z_{\alpha}} = [(C_z)_{\alpha+\Delta\alpha} - (C_z)_{\alpha}] / \Delta\alpha \quad (\text{Normal})$$

$$C_{m_{\alpha}} = [(C_m)_{\alpha+\Delta\alpha} - (C_m)_{\alpha}] / \Delta\alpha \quad (\text{Pitch})$$

$$C_{y_{\beta}} = [(C_y)_{\beta+\Delta\beta} - (C_y)_{\beta}] / \Delta\beta \quad (\text{Side})$$

$$C_{n_{\beta}} = [(C_n)_{\beta+\Delta\beta} - (C_n)_{\beta}] / \Delta\beta \quad (\text{Yaw})$$

$$C_{l_{\beta}} = [(C_l)_{\beta+\Delta\beta} - (C_l)_{\beta}] / \Delta\beta \quad (\text{Roll})$$

The damping derivatives due to vehicle rotation rate are obtained in a similar manner

$$C_{m_q}^{\wedge} = \left\{ [(C_m)_{q+\Delta q} - (C_m)_q] / \Delta q \right\} / [(\dot{c}) / (2V)]$$

etc.

Similarly the control surface derivatives are

$$C_{L_{\delta}} = [(C_L)_{\delta+\Delta\delta} - (C_L)_{\delta}] / \Delta\delta$$

$$C_{m_{\delta}} = [(C_m)_{\delta+\Delta\delta} - (C_m)_{\delta}] / \Delta\delta$$

$$C_{l_{\delta}} = [(C_l)_{\delta+\Delta\delta} - (C_l)_{\delta}] / \Delta\delta$$

$$C_{Y_{\delta}} = [(C_Y)_{\delta+\Delta\delta} - (C_Y)_{\delta}] / \Delta\delta$$

etc.

It is the last term in the numerator of these definitions that are being calculated and printed in the program output.

CONCLUSIONS

An aerodynamic configuration evaluation program has been developed and implemented on a time sharing system with an interactive graphics terminal to maximize responsiveness to the preliminary analysis problem.

The solution is based on potential theory with edge considerations at subsonic/supersonic speeds and impact type finite element analysis at hypersonic conditions. Three-dimensional configurations having multiple non-planar surfaces of arbitrary planform and bodies of non-circular contour may be analyzed. Static, rotary, and control longitudinal and lateral-directional characteristics may be generated.

IBM 3081 computation time of less than one minute of CPU/Mach number at subsonic, supersonic or hypersonic conditions for a typical simulation indicates that the program provides an efficient analysis for systematically performing various aerodynamic configuration tradeoff and evaluation studies. PRIME 850 and VAX 11/780 computation times are approximately fifteen times longer.

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APPENDIX A

SUBSONIC/SUPERSONIC FINITE ELEMENT DERIVATIONS

VELOCITY PERTURBATION POTENTIAL

The velocity potential for a point source can be used to obtain expressions for the velocity potential induced by source and vortex finite elements. Integrations are carried out in the $z=0$ plane and coordinate transformations are used to obtain expressions for constant source and doublet panels having arbitrary orientation. Consider a surface S having a unit normal,

$$\vec{n} = n_x \vec{e}_x + n_y \vec{e}_y + n_z \vec{e}_z$$

where $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ are the unit vectors in the (x,y,z) coordinate system. The velocity potential for a source located at the point (x_0, y_0, z_0) on S is given by the expression,

$$\phi_s(x,y,z) = -\frac{k\sigma}{4\pi R} \quad ; \quad R^2 = (x-x_0)^2 + \beta^2 [(y-y_0)^2 + (z-z_0)^2]$$

therefore $\square^2 \phi_s = 0$, where $\square^2 = (1-M_\infty^2) \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

The velocity potentials induced by a distribution of sources on the surface S is derived more easily if we transform variables to a coordinate system $(\hat{x}, \hat{y}, \hat{z})$ which has the source distribution on the $\hat{z} = 0$ plane. This transformation should also preserve the governing differential equation. First rotate the coordinate system by ϕ to eliminate the y component of the normal.

$$\sin \phi = \frac{-n_y}{\left[n_y^2 + n_z^2 \right]^{1/2}} \quad \cos \phi = \frac{n_z}{\left[n_y^2 + n_z^2 \right]^{1/2}}$$

In the resulting coordinate system the normal will have components

$$(-\sin \alpha, 0, \cos \alpha)$$

where $n_x = -\sin \alpha$

$$n_y = -\cos \alpha \sin \phi$$

$$n_z = \cos \alpha \cos \phi$$

$$\cos^2 \alpha = n_y^2 + n_z^2$$

There finally results the following change of variables

$$\hat{x} = \frac{\beta^2 \sin \alpha [(z-z_0) \cos \phi - (y-y_0) \sin \phi] + (x-x_0) \cos \alpha}{[\cos^2 \alpha + \beta^2 \sin^2 \alpha]^{1/2}}$$

$$\hat{y} = (z-z_0) \sin \phi + (y-y_0) \cos \phi$$

$$\hat{z} = \frac{\cos \alpha [(z-z_0) \cos \phi - (y-y_0) \sin \phi] - (x-x_0) \sin \alpha}{[\cos^2 \alpha + \beta^2 \sin^2 \alpha]^{1/2}}$$

or written in terms of the panel normal,

$$\hat{x} = \frac{- [(y-y_0) n_y + (z-z_0) n_z] \beta^2 n_x + (x-x_0) [n_y^2 + n_z^2]^{1/2}}{[n_y^2 + n_z^2]^{1/2} [\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}}$$

$$\hat{y} = \frac{[(y-y_0) n_z - (z-z_0) n_y]}{[n_y^2 + n_z^2]^{1/2}}$$

$$\hat{z} = \frac{[(y-y_0) n_y + (z-z_0) n_z] + (x-x_0) n_x}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}}$$

If the points (x,y,z) , and (x_0,y_0,z_0) both lie in the plane S, then a vector joining these points must be perpendicular to \vec{n} ,

$$(x-x_0) n_x + (y-y_0) n_y + (z-z_0) n_z = 0$$

and therefore the points lie in the plane, $\hat{z} = 0$. This transformation preserves the governing Prandtl-Glauert differential equation, since we can write,

$$(1-M_\infty^2) \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = (1-M_\infty^2) \frac{\partial^2}{\partial \hat{x}^2} + \frac{\partial^2}{\partial \hat{y}^2} + \frac{\partial^2}{\partial \hat{z}^2}$$

and if again the point (x_0,y_0,z_0) lies on S, for any (x,y,z) ,

$$\hat{x}^2 + \beta^2 (\hat{y}^2 + \hat{z}^2) = (x-x_0)^2 + \beta^2 [(y-y_0)^2 + (z-z_0)^2]$$

The velocity potential for an area in the $\hat{z} = 0$ plane having constant source density is obtained by integrating the influence of infinitesimal source elements over the area. Dropping the $\hat{}$ and using transformed coordinates we have,

$$\Phi_S(x,y,z) = - \frac{k\sigma}{4\pi} \iint_S \frac{1}{R} dx_0 dy_0 \quad R^2 = (x-x_0)^2 + \beta^2 [(y-y_0)^2 + z^2]$$

and since $\nabla^2 \phi_S = 0$, we can say $\nabla^2 \Phi_S = 0$

A doublet at x_0, y_0, z_0 is the derivative of a point source:

$$\phi_D(x,y,z) = \frac{\partial}{\partial z_0} \phi_S(x,y,z-z_0) = - \frac{k\sigma\beta^2 (z-z_0)}{4\pi R^3} = \frac{k\mu (z-z_0)}{4\pi R^3}$$

Integrating from $x_0 = \xi_0$ to infinity yields the potential for a line doublet or elementary horseshoe vortex.

$$\phi_H(x,y,z) = \int_{\xi_0}^{\infty} \phi_D dx_0 = \frac{k\mu}{4\pi} \frac{z}{[(y-y_0)^2 + z^2]} \left[(2-k) + \frac{(x-\xi_0)}{R} \right]$$

$$z_0 = 0$$

And an area of constant vortex strength is obtained by integrating this expression over the panel area:

$$\Phi_\Gamma(x,y,z) = \frac{k\mu}{4\pi} \iint_S \frac{z}{[(y-y_0)^2 + z^2]} \left[(2-k) + \frac{(x-x_0)}{R} \right] dx_0 dy_0$$



The solution of these integrals is performed in the following sections. All integrals may be checked using tables 1 and 2 at the end of this Appendix.

The velocity expressions may be obtained by differentiating the velocity potentials using table 1.

SOURCE PANELS

First the integration is performed over the panels in the x_0 direction as shown in figure 1.

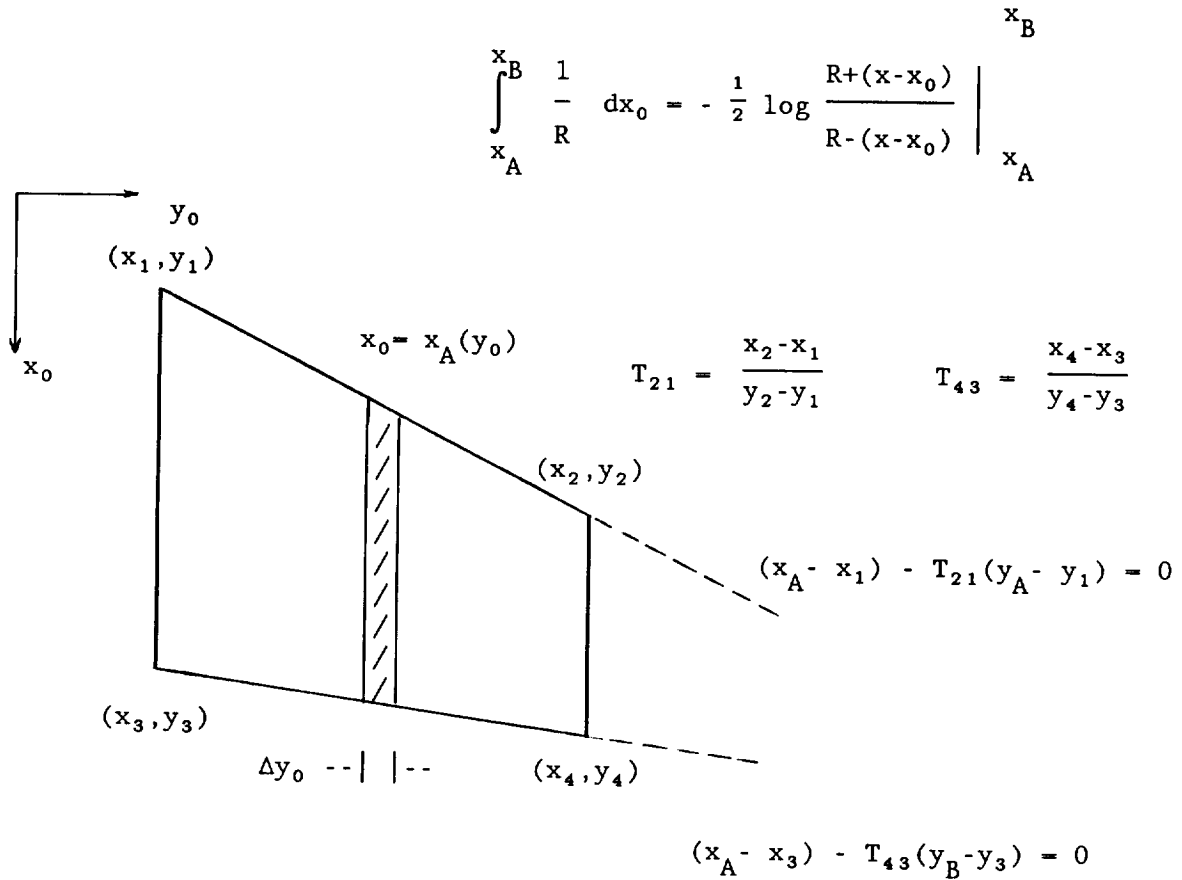


Figure 1. Integration Over Panels in x_0 Direction.

To integrate with respect to y_0 a change of variables is introduced:

$$\begin{aligned} \xi &= (x - x_0) - T(y - y_0) & (x - x_0) &= \xi + \eta \\ \eta &= T(y - y_0) & (y - y_0) &= \frac{1}{T} \eta \\ \zeta &= Tz & z &= \frac{1}{T} \zeta \end{aligned}$$

when $x_0 = x_B$; $T = T_{43}$

$$\xi = (x - x_B) - T(y - y_0) = (x - x_3) - T_B(y - y_3)$$

which is independent of y_0 .

Therefore using $T^2 b^2 = T^2 + \beta^2$, and integrating with ξ constant

$$\begin{aligned} \int_{y_1}^{y_2} \frac{1}{2} \log \frac{R+(x-x_0)}{R-(x-x_0)} dy_0 &= \frac{1}{T} \int_{y_1}^{y_2} \frac{1}{2} \log \frac{R+(\xi+\eta)}{R-(\xi+\eta)} d\eta \\ &= \frac{1}{T} \left\{ \eta \frac{1}{2} \log \frac{R+(\xi+\eta)}{R-(\xi+\eta)} + \xi \frac{1}{b} \frac{1}{2} \log \frac{bR+(\xi+B^2\eta)}{bR+(\xi+B\eta)} + \zeta \tan^{-1} \frac{\zeta R}{\xi\eta-\zeta^2} \right\}_{\eta_1}^{\eta_2} \end{aligned}$$

This integration may be checked using table 2.

Each of the four integration limits corresponds to a corner of the quadrilateral. Placing the origin of the x_0, y_0 coordinate system at one corner, and setting $B^2 = T^2 + \beta^2$, the contribution to Φ becomes:

$$\begin{aligned} \phi(x,y,z,T) = - \frac{\sigma k}{4\pi} \left\{ y \frac{1}{2} \log \frac{R+x}{R-x} + (x-Ty) \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} \right. \\ \left. + z \tan^{-1} \frac{zR}{xy-Tr} \right\} \end{aligned}$$

and combining each of the four corners:

$$\begin{aligned} \Phi(x,y,z,T) = & \phi(x-x_1, y-y_1, z, T_{21}) - \phi(x-x_2, y-y_2, z, T_{21}) \\ & - \phi(x-x_3, y-y_3, z, T_{43}) + \phi(x-x_4, y-y_4, z, T_{43}) \end{aligned}$$

VORTEX PANELS

Analogous to the source panels the integration is first performed in the x_0 direction.

$$\int_{x_A}^{x_B} \frac{z}{[(y-y_0)^2 + z^2]} \left[(2-k) + \frac{(x-x_0)}{R} \right] dx_0 = \left\{ \frac{-z [(2-k)(x-x_0) + R]}{[(y-y_0)^2 + z^2]} \right\}_{x_0=x_A}^{x_0=x_B}$$

changing variables and integrating with respect to η

$$\begin{aligned}
& - \int_{y_1}^{y_2} \frac{-z [(2-k)(x-x_0) + R]}{[(y-y_0)^2 + z^2]} dy_0 = \int_{\eta_1}^{\eta_2} \frac{\zeta [(2-k)(\xi+\eta) + R]}{\eta^2 + \zeta^2} d\eta \\
& = (2-k) \left\{ \xi \tan^{-1} \frac{\eta}{\zeta} + \zeta \frac{1}{2} \log (\eta^2 + \zeta^2) \right\}_{\eta_1}^{\eta_2} \\
& + \left\{ -\zeta \frac{1}{2} \log \frac{R+(\xi+\eta)}{R-(\xi+\eta)} + b^2 \zeta \frac{1}{B} \frac{1}{2} \log \frac{bR+(\xi+B^2\eta)}{bR-(\xi+B^2\eta)} - \xi \tan^{-1} \frac{\zeta R}{\xi\eta - \zeta^2} \right\}_{\eta_2}^{\eta_1}
\end{aligned}$$

therefore for one corner or integration limit

$$\begin{aligned}
\phi(x,y,z,T) = & \frac{kCp}{8\pi} \left\{ Tz \frac{1}{2} \log \frac{R+x}{R-x} - z B^2 \frac{1}{B} \frac{1}{2} \log \frac{BR+(Tx+\beta^2 y)}{BR-(Tx+\beta^2 y)} \right. \\
& \left. + (x-Ty) \tan^{-1} \frac{zR}{xy-Tr} - (2-k) \left[Tz \frac{1}{2} \log r^2 + (x-Ty) \tan^{-1} \frac{y}{z} \right] \right\}
\end{aligned}$$

and

$$\begin{aligned}
\Phi(x,y,z,T) = & \phi(x-x_1, y-y_1, z, T_{21}) - \phi(x-x_2, y-y_2, z, T_{21}) \\
& - \phi(x-x_3, y-y_3, z, T_{43}) + \phi(x-x_4, y-y_4, z, T_{43})
\end{aligned}$$

Velocity Component Transformations

The velocity expressions may be obtained by differentiating the velocity potentials. The results of this are given on page 33. Since all integrations were done in the $\hat{}$ coordinate system, we must consider the variable transformation to obtain the actual perturbation velocities.

$$\begin{aligned}
u = \frac{\partial}{\partial x} \phi(x,y,z) &= \frac{\hat{u} \cos \alpha - \hat{w} \sin \alpha}{[\cos^2 \alpha + \beta^2 \sin^2 \alpha]^{1/2}} \\
v = \frac{\partial}{\partial y} \phi(x,y,z) &= \frac{-[\beta^2 \hat{u} \sin \alpha + \hat{w} \cos \alpha] \sin \phi}{[\cos^2 \alpha + \beta^2 \sin^2 \alpha]^{1/2}} + \hat{v} \cos \phi \\
w = \frac{\partial}{\partial z} \phi(x,y,z) &= \frac{[\beta^2 \hat{u} \sin \alpha + \hat{w} \cos \alpha] \cos \phi}{[\cos^2 \alpha + \beta^2 \sin^2 \alpha]^{1/2}} + \hat{v} \sin \phi
\end{aligned}$$

We can also write this in terms of the panel normal,

$$\begin{aligned}
 u &= \frac{\hat{u} (n_y^2 + n_z^2)^{1/2} + \hat{w} n_x}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}} \\
 v &= \frac{[-\beta^2 \hat{u} n_x + \hat{w} (n_y^2 + n_z^2)^{1/2}] n_y}{(n_y^2 + n_z^2)^{1/2} [\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}} + \frac{\hat{v} n_z}{(n_y^2 + n_z^2)^{1/2}} \\
 w &= \frac{[-\beta^2 \hat{u} n_x + \hat{w} (n_y^2 + n_z^2)^{1/2}] n_z}{(n_y^2 + n_z^2)^{1/2} [\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}} - \frac{\hat{v} n_y}{(n_y^2 + n_z^2)^{1/2}}
 \end{aligned}$$

where

$$\hat{u} = \frac{\partial}{\partial x} \hat{\phi}(x, y, z) \quad \hat{v} = \frac{\partial}{\partial y} \hat{\phi}(x, y, z) \quad \hat{w} = \frac{\partial}{\partial z} \hat{\phi}(x, y, z)$$

The derivatives of the velocity potential expressions may be obtained by using table 1, and are given on page 33.

VERIFICATION OF THE PERTURBATION VELOCITY EXPRESSIONS

To establish that these are the correct perturbation velocities the following criteria must be met:

1. Laplace's equation must be satisfied

$$\beta^2 \Phi_{xx} + \Phi_{yy} + \Phi_{zz} = 0$$

or the equivalent

$$u_y = v_x$$

$$u_z = w_x$$

$$v_z = w_y$$

$$\beta^2 u_x + v_y + w_z = 0$$

2. The correct discontinuity or jump in the perturbation velocity must occur at the surface of the quadrilateral panel area. For the source panel the jump occurs in the normal or w velocity and on the vortex panel there must be a jump of constant magnitude in the u perturbation velocity over the panel area. The perturbation velocities should be continuous elsewhere, except on the trailing vortex sheet of the vortex panel.
3. The perturbation velocities must go to zero as upstream infinity is approached.
4. For the vortex panel the trailing vorticity must extend straight back to downstream infinity. This means that any discontinuity in the v velocity must be zero outside the spanwise boundaries of the panel and must be zero upstream of the panel.

The first criteria can be established by using the derivatives given in table 1.

The second criteria can be established by noting that all terms except

$$\tan^{-1} \frac{z R}{xy - T(y^2 + z^2)} \quad \text{and} \quad \tan^{-1} \frac{y}{z}$$

are continuous at $z = 0$. Consider these terms keeping in mind that the contributions from all four corners must be included.

If we let

$$\xi = (x - x_1) - T(y - y_1) \quad \text{and} \quad \eta = (x - x_2) - T(y - y_2)$$

$$R_1^2 = (x - x_1)^2 + \beta^2 [(y - y_1)^2 + z^2]$$

and use

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A + B}{1 - AB}$$

then the contributions from both corners on the leading edge can be combined as follows.

$$\begin{aligned} f(z) &= \tan^{-1} \frac{z R_1}{\xi(y - y_1) - Tz} - \tan^{-1} \frac{z R_2}{\xi(y - y_2) - Tz} \\ &= \tan^{-1} \frac{z \{ [\xi(y - y_2) - Tz]^2 R_1 - [\xi(y - y_1) - Tz]^2 R_2 \}}{[\xi(y - y_1) - Tz]^2 [\xi(y - y_2) - Tz]^2 + z^2 R_1 R_2} \end{aligned}$$

If we define

$$\text{sgn}(z) = \begin{cases} 1 & z > 0 \\ -1 & z < 0 \end{cases}$$

$$\lim_{a \rightarrow 0} [\tan^{-1} \frac{a}{b}] = \begin{cases} 0 & b > 0 \\ \pi \text{sgn}(a) & b < 0 \end{cases} \quad -\pi \leq \tan^{-1}(c) \leq \pi$$

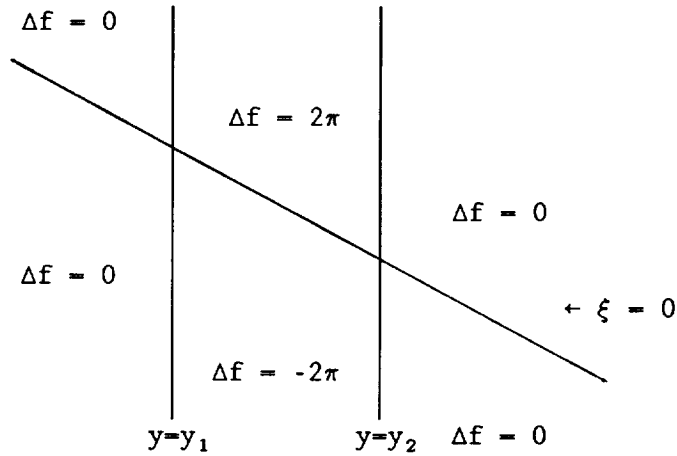
and

$$\lim_{z \rightarrow 0} \left\{ \tan^{-1} \frac{z R_1}{\xi(y-y_1)-Tz} - \tan^{-1} \frac{z R_2}{\xi(y-y_2)-Tz} \right\} = \lim_{z \rightarrow 0} f(z)$$

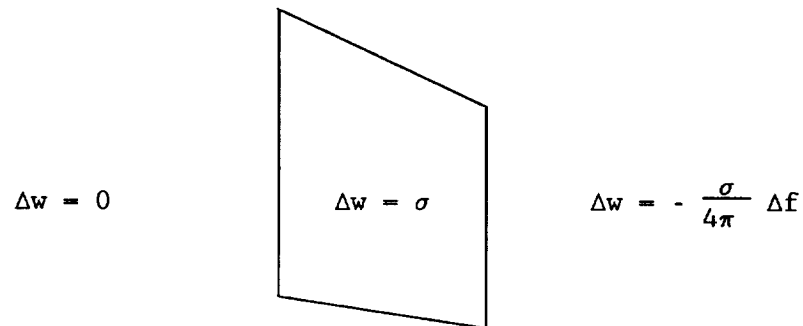
$$= \begin{cases} 0 & (y-y_1)(y-y_2) > 0 \\ -\pi \text{sgn}(z)\text{sgn}(\xi) & (y-y_1)(y-y_2) < 0 \end{cases}$$

The discontinuity, or jump, in $f(z)$ at $z = 0$ becomes,

$$\Delta f(z) = \lim_{z \rightarrow 0^+} [f(z)] - \lim_{z \rightarrow 0^-} [f(z)]$$



Therefore when a similar procedure is carried out for the trailing edge of a source panel and we subtract the results, we obtain the following jump in the w perturbation velocity.

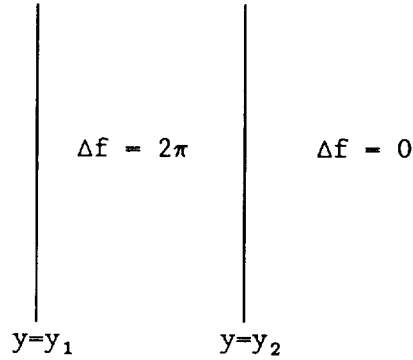


For the vortex panel (subsonic) we have an additional term. Considering both additional terms from the leading edge corners:

$$f(z) = \tan^{-1} \frac{(y-y_1)}{z} - \tan^{-1} \frac{(y-y_2)}{z} = \tan^{-1} \frac{z(y_2-y_1)}{(y-y_1)(y-y_2)+z^2}$$

$$\lim_{z \rightarrow 0} \left\{ \tan^{-1} \frac{(y-y_1)}{z} - \tan^{-1} \frac{(y-y_2)}{z} \right\} = \begin{cases} 0 & (y-y_1)(y-y_2) > 0 \\ \pi \operatorname{sgn}(z) & (y-y_1)(y-y_2) < 0 \end{cases}$$

$$\Delta \left\{ \tan^{-1} \frac{(y-y_1)}{z} - \tan^{-1} \frac{(y-y_2)}{z} \right\} = \begin{cases} 0 & (y-y_1)(y-y_2) > 0 \\ 2\pi & (y-y_1)(y-y_2) < 0 \end{cases}$$



Therefore combining the terms

$$\begin{aligned} \Delta \left\{ \tan^{-1} \frac{z R_1}{\xi(y-y_1)-Tz} - \tan^{-1} \frac{z R_2}{\xi(y-y_2)-Tz} - \tan^{-1} \frac{y-y_1}{z} - \tan^{-1} \frac{y-y_2}{z} \right\} \\ = \Delta f(z) = \lim_{z \rightarrow 0^+} [f(z)] - \lim_{z \rightarrow 0^-} [f(z)] \\ = \begin{cases} 0 & (y-y_1)(y-y_2) > 0, \text{ or } \xi < 0 \\ -4\pi & \text{otherwise} \end{cases} \end{aligned}$$

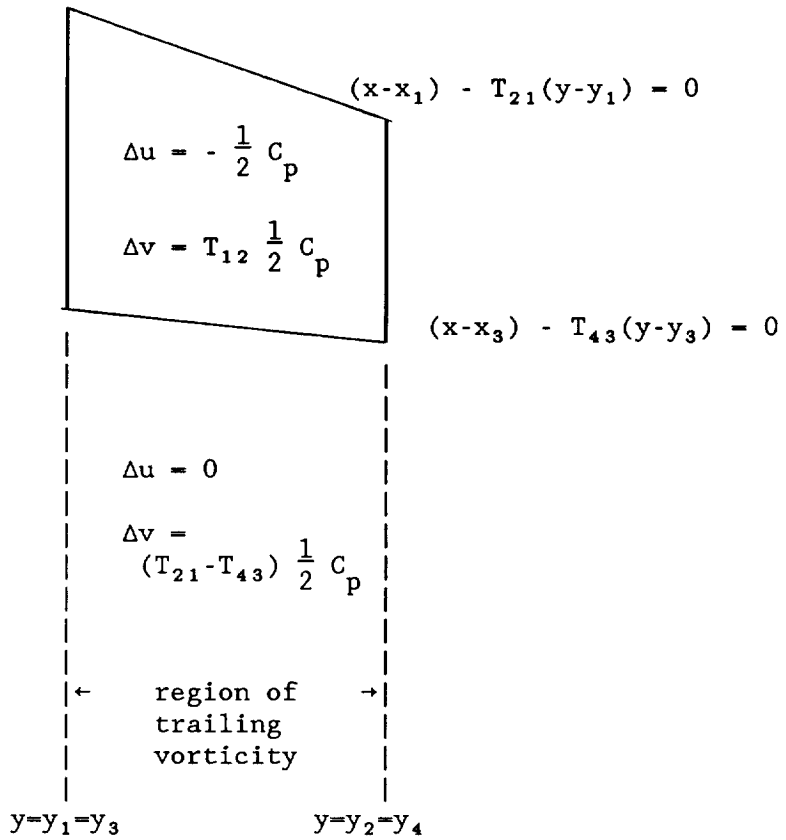
Since contribution from each panel corner is:

$$\begin{aligned} \Delta u &= \frac{C_p}{8\pi} \Delta \left\{ \tan^{-1} \frac{z R}{xy-T(y^2+z^2)} - \tan^{-1} \frac{y}{z} \right\} \\ \Delta v &= -T \frac{C_p}{8\pi} \Delta \left\{ \tan^{-1} \frac{z R}{xy-T(y^2+z^2)} - \tan^{-1} \frac{y}{z} \right\} \end{aligned}$$

after summing all four panels corners (both edges), we obtain the following for Δu and Δv

$$\Delta u = 0$$

$$\Delta v = 0$$



To verify the third criteria we must show that all of the functions approach zero when all four corners are considered as $x \rightarrow -\infty$

$$\begin{aligned} \frac{1}{2} \log \frac{R+x}{R-x} - \frac{1}{2} \log (y^2 + z^2) &= \frac{1}{2} \log \frac{\beta^2 (y^2 + z^2)}{(R-x)^2} - \frac{1}{2} \log (y^2 + z^2) \\ &= -\frac{1}{2} \log (R-x)^2 - \frac{1}{2} \log \beta^2 \end{aligned}$$

Therefore considering both corners on the leading edge of the panel

$$\lim_{x \rightarrow -\infty} [R] = |x|$$

$$\lim_{x \rightarrow -\infty} \left\{ \frac{1}{2} \log \frac{R_1 + (x - x_1)}{R_1 - (x - x_1)} - \frac{1}{2} \log \frac{R_2 + (x - x_2)}{R_2 - (x - x_2)} \right\} = \lim_{x \rightarrow -\infty} \left\{ \frac{1}{2} \log \frac{(x - x_2)^2}{(x - x_1)^2} \right\} = 0$$

$$\lim_{x \rightarrow -\infty} \left\{ \frac{R_1 + (x - x_1)}{(y - y_1)^2 + z^2} \right\} = 0$$

$$\lim_{x \rightarrow -\infty} \left\{ \frac{1}{B} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} \right\} = \frac{1}{2} \log \frac{B + T}{B - T}$$

and therefore this limit is also zero when both corners of the leading or trailing edges are considered. Since all terms are accounted for, the perturbation velocities are zero far upstream.

$$\text{Since } B^2 R^2 - (Tx + \beta^2 y)^2 = \beta^2 [(x - Ty)^2 + B^2 z^2]$$

$$\log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} = \log \frac{BR + (Tx + \beta^2 y)}{\beta^2 [(x - Ty)^2 + B^2 z^2]} = \log \frac{\beta^2 [(x - Ty)^2 + B^2 z^2]}{BR - (Tx + \beta^2 y)}$$

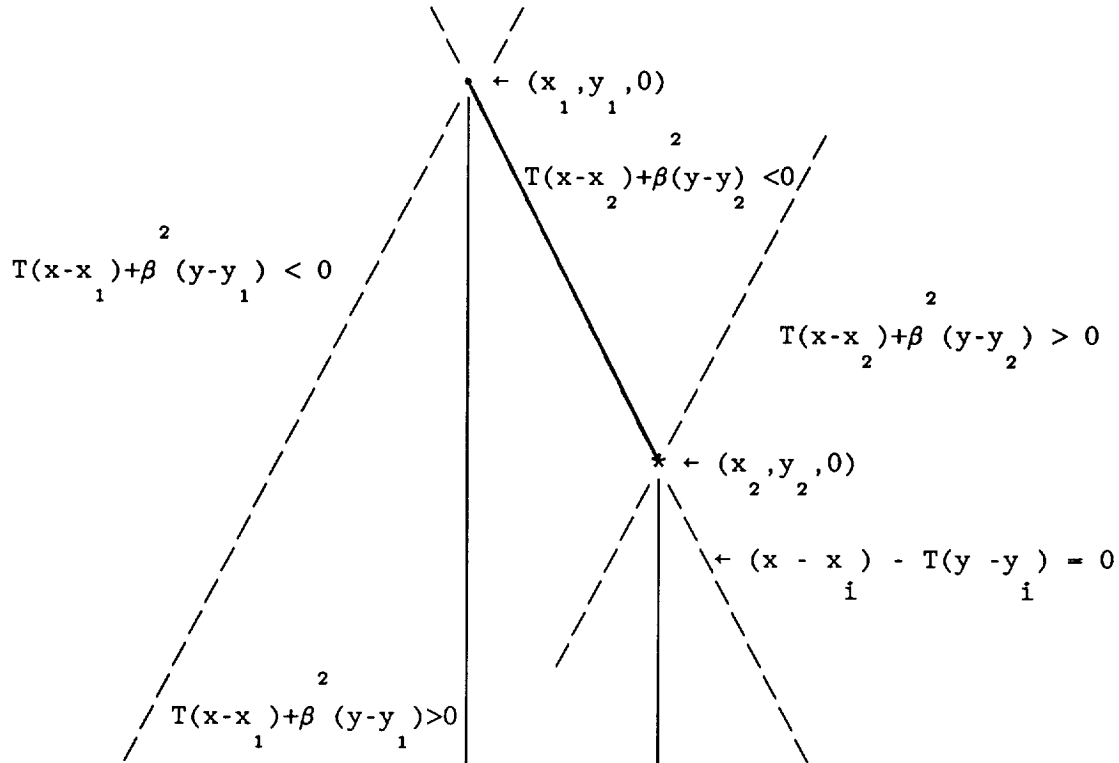
there is an apparent singularity along the line

$$(x - Ty) = 0, \quad z = 0$$

However this singularity may be removed by combining the contributions from both corners of the leading or trailing edges of the panel. Along either of these edges the values of

$$(x - x_i) - T(y - y_i) \quad \text{and} \quad z$$

are the same for each of the panel corners.



It can be seen from the above diagram that $(Tx + \beta^2 y)$ will have the same sign on a point $(x, y, 0)$ which lies outside the spanwise boundaries of the quadrilateral. Therefore outside the spanwise boundaries the term

$$\log [(x-Ty)^2 + B^2 z^2]$$

can be canceled by combining both corners, and the resulting term

$$\pm \frac{1}{B} \log \frac{BR_1 \pm [T(x-x_1) + \beta^2 (y-y_1)]}{BR_2 \pm [T(x-x_2) + \beta^2 (y-y_2)]}$$

will not be singular if the correct + or - sign is chosen. Within the spanwise boundary an actual singularity occurs on the panel edge.

The term $\frac{1}{2} \log \frac{R+x}{R-x}$ also has a possible singularity. This term can be written

$$\frac{1}{2} \log \frac{R+x}{R-x} = \frac{1}{2} \log \frac{(R+x)^2}{\beta^2 (y+z)^2}$$

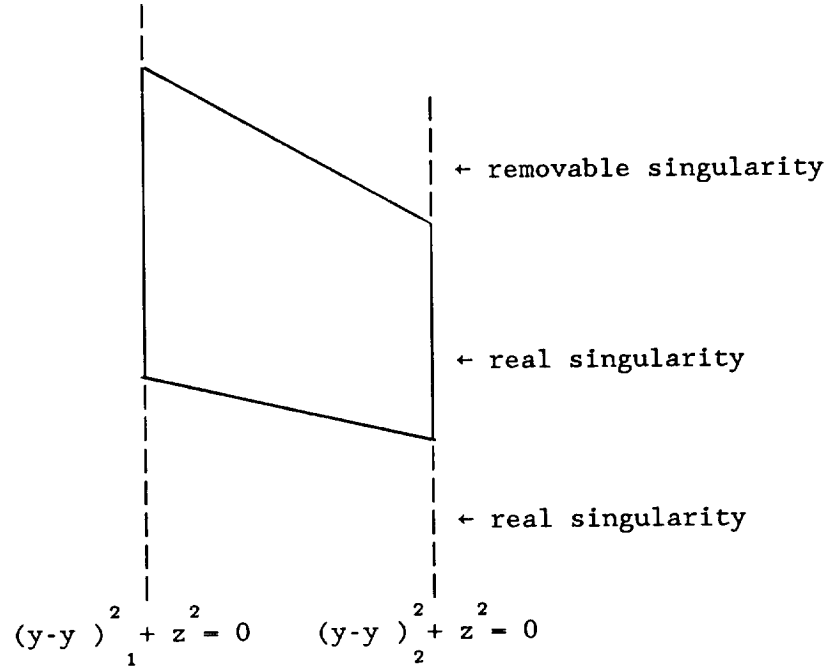
For the source panel the singularity may be removed for points along

$y^2 + z^2 = 0$ which are outside of the panel boundaries.

If $(x-x_1)$ and $(x-x_3)$ have the same sign the combination of the two terms gives

$$\frac{1}{2} \log \frac{R_1+(x-x_1)}{R_1-(x-x_1)} - \frac{1}{2} \log \frac{R_3+(x-x_3)}{R_3-(x-x_3)} = \pm \frac{1}{2} \log \frac{R_1 \pm (x-x_1)}{R_3 \pm (x-x_3)}$$

where the correct sign is chosen to remove the singularity. On the panel edge the singularity is real and cannot be removed.



For a vortex panel the terms (subsonic)

$$\frac{1}{2} \log \frac{R+x}{R-x} - \frac{1}{2} \log(y^2 + z^2) \quad \text{and} \quad \frac{y(R+x)}{(y+z)^2}$$

Both have real singularities for $x > 0$ (downstream) and removable singularities for $x < 0$ (upstream). The real singularities occur on the panel edges and on the edge of the trailing vortex sheet.

as $r^2 = (y^2 + z^2) \rightarrow 0$, and $(x-x_1) < 0$ (upstream edge extension)

$$R_1 = \left[(x-x_1)^2 + \beta^2 r^2 \right]^{1/2} \rightarrow -(x-x_1) \left[1 + \beta^2 \frac{r^2}{(x-x_1)^2} \right]$$

and combining the contributions from leading and trailing edges,

$$\lim_{r^2 \rightarrow 0} \left[\frac{y [R_1+(x-x_1)]}{r^2} - \frac{y [R_3+(x-x_3)]}{r^2} \right] = -\frac{\beta^2 y}{2} \frac{(x_3-x_1)}{(x-x_3)(x-x_1)} = 0$$

VELOCITY FLUX FROM AN INCLINED (BODY) SOURCE PANEL

The perturbation velocity normal to a panel surface is given by the expression

$$\begin{aligned}
 \vec{u} \cdot \vec{n} &= u n_x + v n_y + w n_z \\
 &= \frac{\hat{u} n_x (n_y^2 + n_z^2)^{1/2} + \hat{w} n_x^2}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}} \\
 &+ \frac{[-\beta^2 \hat{u} n_x + \hat{w} (n_y^2 + n_z^2)^{1/2}] n_y^2}{(n_y^2 + n_z^2)^{1/2} [\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}} + \frac{\hat{v} n_z n_y}{(n_y^2 + n_z^2)^{1/2}} \\
 &+ \frac{[-\beta^2 \hat{u} n_x + \hat{w} (n_y^2 + n_z^2)^{1/2}] n_z^2}{(n_y^2 + n_z^2)^{1/2} [\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}} - \frac{\hat{v} n_y n_z}{(n_y^2 + n_z^2)^{1/2}} \\
 &= \frac{\hat{w} + \hat{u} [1 - \beta^2] n_x (n_y^2 + n_z^2)^{1/2}}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}}
 \end{aligned}$$

Since across the panel surface $\Delta \hat{u} = 0$, the rate of outflow from the panel surface is given by,

$$= \frac{\Delta \hat{w}}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}}$$

and since $\Delta \hat{w} = \sigma$

across the panel surface

$$\vec{u}_+ \cdot \vec{n}_+ + \vec{u}_- \cdot \vec{n}_- = \frac{\sigma}{[\beta^2 n_x^2 + n_y^2 + n_z^2]^{1/2}}$$

where + and - signify the upper and lower surfaces.

SUPERSONIC VELOCITIES - SPECIAL CONSIDERATIONS

The velocity perturbation influence equations for supersonic flows are treated by taking only the real parts of the expressions. This means that

$R = [x^2 + \beta^2(y^2 + z^2)]^{1/2}$ is set equal to zero for points which lie outside the downstream Mach cone from any given corner. Therefore, R and

$\frac{1}{2} \log \frac{R+x}{R-x}$ are zero for points which lie outside the downstream Mach cone.

For $B^2 = (T^2 + \beta^2) > 0$, there are no problems using this method.

If $B^2 = (T^2 + \beta^2) < 0$, and $\left[T^2 + \beta^2 \right]^{1/2} = i B$ where $i = \sqrt{-1}$

$$\frac{1}{i B} \frac{1}{2} \log \frac{(Tx + \beta^2 y) + i B R}{(Tx + \beta^2 y) - i B R} = \frac{1}{B} \tan^{-1} \frac{B R}{(Tx + \beta^2 y)}$$

and combining two corners,

$$\begin{aligned} F_2 &= \frac{1}{B} \frac{1}{2} \log \frac{[T(x-x_1) + \beta^2(y-y_1)] + B R_1}{[T(x-x_1) + \beta^2(y-y_1)] - B R_1} - \frac{1}{B} \frac{1}{2} \log \frac{[T(x-x_2) + \beta^2(y-y_2)] + B R_2}{[T(x-x_2) + \beta^2(y-y_2)] - B R_2} \\ &= \frac{1}{B} \tan^{-1} \frac{B \left\{ [T(x-x_2) + \beta^2(y-y_2)] R_1 - [T(x-x_1) + \beta^2(y-y_1)] R_2 \right\}}{[T(x-x_2) + \beta^2(y-y_2)][T(x-x_1) + \beta^2(y-y_1)] - B^2 R_1 R_2} \end{aligned}$$

If $z = 0$ and either R_1 or R_2 is zero and we allow the other to approach zero, the value of F_2 becomes

$$F_2 = \begin{cases} \frac{\pi}{B} & [T(x-x_1) - \beta^2(y-y_1)][T(x-x_2) - \beta^2(y-y_2)] < 0 \\ 0 & [T(x-x_1) - \beta^2(y-y_1)][T(x-x_2) - \beta^2(y-y_2)] > 0 \end{cases}$$

Therefore if R_1 and R_2 are zero but we are inside the envelope of Mach cones from the leading edge (see figure 2), the value of F_2 is set equal to

$$F_2 = \frac{\pi}{B} \quad \text{if} \quad \begin{cases} [T(x-x_1) - \beta^2(y-y_1)][T(x-x_2) - \beta^2(y-y_2)] < 0 \\ R_1^2 < 0 \quad R_2^2 > 0 \\ (x-Ty)^2 > (\beta^2 - T^2)z^2 \end{cases}$$

$$T = \tan \Lambda \quad [(x-x_1) - T(y-y_1)]^2 = [(x-x_2) - T(y-y_2)]^2 > (\beta^2 - T^2)z^2$$

$$\beta^2 - T^2 > 0 \quad (x-x_1)^2 < \beta^2 [(y-y_1)^2 + z^2]$$

$$(x-x_2)^2 < \beta^2 [(y-y_2)^2 + z^2]$$

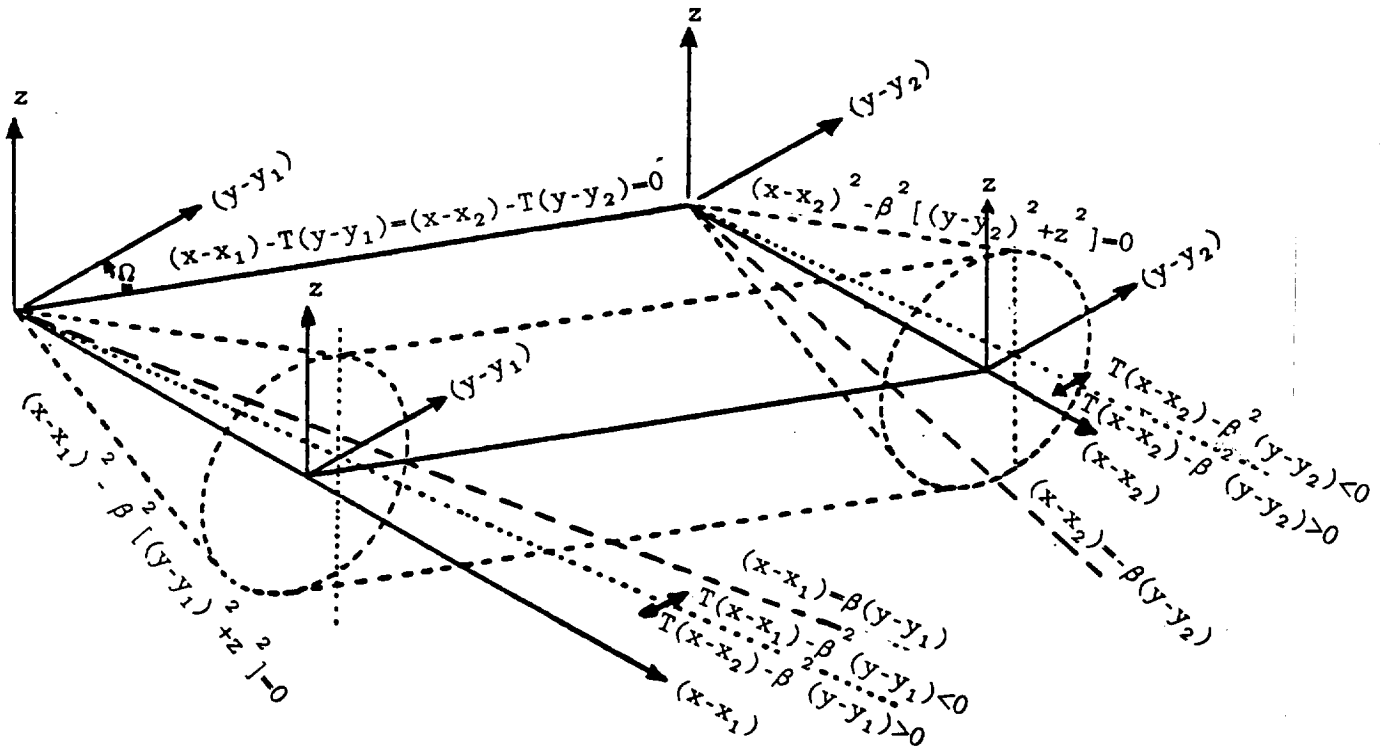


Figure 2b Supersonic Leading Edge Mach Cone Envelope

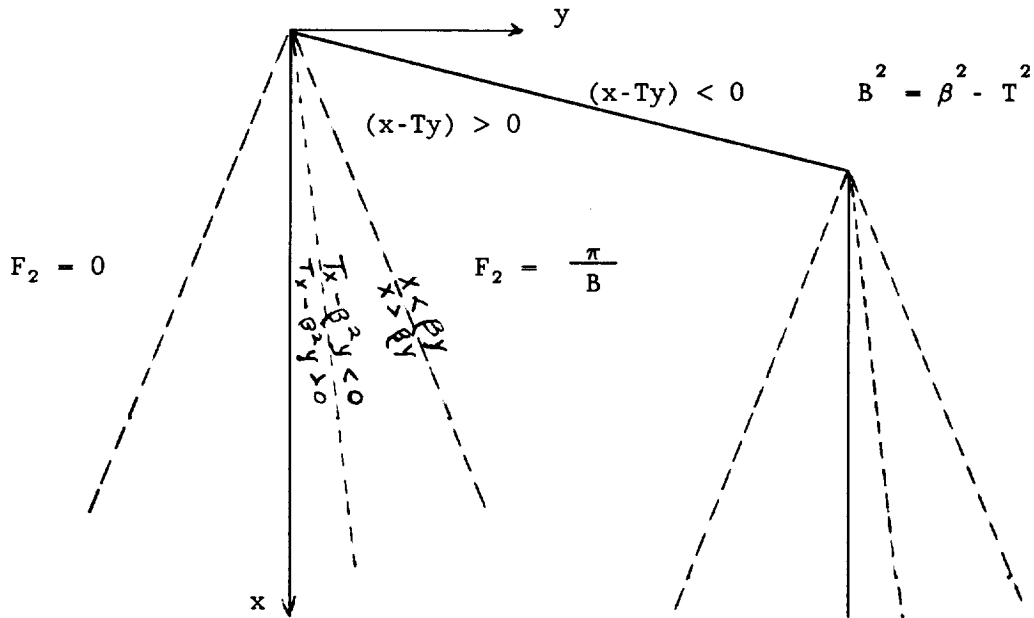


Figure 2b Supersonic Leading Edge Mach Cone Envelope

The intersection of the lines determined by

$$(x-Ty)^2 = (\beta^2 - T^2)z^2$$

and

$$x^2 = \beta^2(y^2 + z^2)$$

occurs on the line $\begin{cases} y = ax \\ z = bx \end{cases}$

therefore $1 = \beta^2(a^2 + b^2)$

$$1 - 2aT + T^2 a^2 = (\beta^2 - T^2)b^2$$

$$\beta^2(1 - 2aT + T^2 a^2) = (1 - \beta^2 a^2)(\beta^2 - T^2) = \beta^2(\beta^2 - T^2)b^2$$

$$\beta^4 a^2 - 2\beta^2 Ta + T^2 = 0 \Rightarrow \beta^2 a = \pm T$$

$$\beta^4 b^2 = \beta^2 - T^2 \Rightarrow \beta^2 b = \pm \left[\beta^2 - T^2 \right]^{1/2}$$

therefore the line is determined by

$$Tx - \beta^2 y = 0 ; \quad \beta^2 z^2 = (\beta^2 - T^2)x^2$$

or

$$Tx - \beta^2 y = 0 ; \quad (x-Ty)^2 = (\beta^2 - T^2)z^2$$

As $T \rightarrow \beta$ (sonic leading edge) the value of $(T^2 - \beta^2) \rightarrow 0$. In this case

$$\lim_{T \rightarrow \beta} F_2 = \frac{(R_1 - R_2)}{T [(x - x_1) - T(y - y_1)]} ; \quad [(x - x_1) - T(y - y_1)] > 0$$

SONIC EDGES

As $B^2 = T^2 + \beta^2 \rightarrow 0$, numerical difficulties arise in the evaluation of the function,

$$\frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} = \frac{1}{B} \frac{1}{2} \log \frac{BR + [T(x - Ty) + B^2 y]}{BR - [T(x - Ty) + B^2 y]}$$

However, for small values of B^2 , this function can be easily evaluated numerically by using a few terms of a series expansion. To generate the series, first we set

$$a = T(x - Ty) + B^2 y$$

$$b = (T^2 - B^2) [(x - Ty)^2 + B^2 z^2]$$

$$\delta = \frac{B^2 R^2}{b}$$

and therefore

$$a^2 = B^2 R^2 + b = b(1 + \delta)$$

$$a B^2 R^2 = b^2 \delta (1 + \delta)$$

therefore

$$\begin{aligned} \frac{1}{2} \log \frac{BR + [T(x - Ty) + B^2 y]}{BR - [T(x - Ty) + B^2 y]} &= \frac{1}{2} \log \frac{(1 + \delta)^{1/2} + \delta^{1/2}}{(1 + \delta)^{1/2} - \delta^{1/2}} = \frac{1}{2} \int_0^\delta \frac{dt}{[t(1 + t)]^{1/2}} \\ &= \frac{1}{2} \int_0^\delta \frac{dt}{t^{1/2}} \left[1 - \frac{1}{2} t + \frac{1 \cdot 3}{2^2} \frac{1}{2!} t^2 - \frac{1 \cdot 3 \cdot 5}{2^3} \frac{1}{3!} t^3 + \dots \right] \\ &= \delta^{1/2} \left[1 - \frac{1}{3 \cdot 2} \delta + \frac{1 \cdot 3}{5 \cdot 2^2} \frac{1}{2!} \delta^2 - \frac{1 \cdot 3 \cdot 5}{7 \cdot 2^3} \frac{1}{3!} \delta^3 + \dots \right] \\ &= [\delta(1 + \delta)]^{1/2} \left[1 - \frac{1}{3 \cdot 2} \delta + \frac{1 \cdot 3}{5 \cdot 2^2} \frac{1}{2!} \delta^2 - \frac{1 \cdot 3 \cdot 5}{7 \cdot 2^3} \frac{1}{3!} \delta^3 + \dots \right] \\ &\quad \cdot \left[1 - \frac{1}{2} \delta + \frac{1 \cdot 3}{2^2} \frac{1}{2!} \delta^2 - \frac{1 \cdot 3 \cdot 5}{2^3} \frac{1}{3!} \delta^3 + \dots \right] \end{aligned}$$

$$\begin{aligned}
= & \frac{a B R}{b} \left[1 - \frac{2}{3} \delta + \frac{2^3}{3 \cdot 5} \delta^2 - \frac{2^4}{5 \cdot 7} \delta^3 + \frac{2^7}{5 \cdot 7 \cdot 9} \delta^4 \right. \\
& - \frac{2^8}{7 \cdot 9 \cdot 11} \delta^5 + \frac{2^{10}}{3 \cdot 7 \cdot 11 \cdot 13} \delta^6 - \frac{2^{11}}{5 \cdot 9 \cdot 11 \cdot 13} \delta^7 \\
& + \frac{2^{15}}{5 \cdot 9 \cdot 11 \cdot 13 \cdot 17} \delta^8 - \frac{2^{16}}{5 \cdot 11 \cdot 13 \cdot 17 \cdot 19} \delta^9 \\
& + \frac{2^{18}}{3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19} \delta^{10} - \frac{2^{19}}{3 \cdot 7 \cdot 13 \cdot 17 \cdot 19 \cdot 23} \delta^{11} \\
& + \frac{2^{22}}{7 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 25} \delta^{12} - \frac{2^{23}}{7 \cdot 17 \cdot 19 \cdot 23 \cdot 25 \cdot 27} \delta^{13} \\
& \left. + \cdot \cdot \cdot \right]
\end{aligned}$$

This series converges rapidly for small values of δ , or small values of B^2 .

TABLE 1

TABLE OF DERIVATIVES

$$r^2 = y^2 + z^2, \quad R^2 = x^2 + \beta^2 r^2, \quad \beta^2 = 1 - M_\infty^2, \quad B^2 = T^2 + \beta^2$$

$$R \frac{\partial}{\partial x} R = x$$

$$\frac{\partial}{\partial x} \frac{1}{2} \log \frac{R+x}{R-x} = \frac{1}{R}$$

$$\frac{\partial}{\partial x} \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} = - \frac{1}{R} \frac{xy - Tr^2}{[(x - Ty)^2 + B^2 z^2]}$$

$$\frac{\partial}{\partial x} \tan^{-1} \frac{z R}{xy - Tr^2} = - \frac{1}{R} \frac{z (Tx + \beta^2 y)}{[(x - Ty)^2 + B^2 z^2]}$$

$$R \frac{\partial}{\partial y} R = \beta^2 y$$

$$\frac{\partial}{\partial y} \frac{1}{2} \log \frac{R+x}{R-x} = - \frac{1}{R} \frac{xy}{r^2}$$

$$\frac{\partial}{\partial y} \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} = \frac{1}{R} \frac{x(x - Ty) + \beta^2 z^2}{[(x - Ty)^2 + B^2 z^2]}$$

$$\frac{\partial}{\partial y} \tan^{-1} \frac{z R}{xy - Tr^2} = \frac{1}{R} \frac{Tz (Tx + \beta^2 y)}{[(x - Ty)^2 + B^2 z^2]} - \frac{1}{R} \frac{xz}{r^2}$$

$$R \frac{\partial}{\partial z} R = \beta^2 z$$

$$\frac{\partial}{\partial z} \frac{1}{2} \log \frac{R+x}{R-x} = - \frac{1}{R} \frac{xz}{r^2}$$

$$\frac{\partial}{\partial z} \frac{1}{B} \frac{1}{2} \log \frac{BR + (Tx + \beta^2 y)}{BR - (Tx + \beta^2 y)} = - \frac{1}{R} \frac{z (Tx + \beta^2 y)}{[(x - Ty)^2 + B^2 z^2]}$$

$$\frac{\partial}{\partial z} \tan^{-1} \frac{z R}{xy - Tr^2} = \frac{1}{R} \frac{(x - Ty)(Tx + \beta^2 y)}{[(x - Ty)^2 + B^2 z^2]} + \frac{1}{R} \frac{xy}{r^2}$$

TABLE 2

TABLE OF DERIVATIVES

$$\rho^2 = \eta^2 + \zeta^2, \quad R^2 = (\xi + \eta)^2 + (b^2 - 1)\rho^2, \quad T^2 b^2 = T^2 + \beta^2$$

$$R \frac{\partial}{\partial \xi} R = (\xi + \eta)$$

$$\frac{\partial}{\partial \xi} \frac{1}{2} \log \frac{R + (\xi + \eta)}{R - (\xi + \eta)} = \frac{1}{R}$$

$$\frac{\partial}{\partial \xi} \frac{1}{b} \frac{1}{2} \log \frac{bR + (\xi + b^2 \eta)}{bR - (\xi + b^2 \eta)} = \frac{1}{R} \frac{\zeta^2 - \xi \eta}{[\xi^2 + b^2 \zeta^2]}$$

$$\frac{\partial}{\partial \xi} \tan^{-1} \frac{\zeta R}{\xi \eta - \zeta^2} = - \frac{1}{R} \frac{\zeta (\xi + b^2 \eta)}{[\xi^2 + b^2 \zeta^2]}$$

$$R \frac{\partial}{\partial \eta} R = (\xi + b^2 \eta)$$

$$\frac{\partial}{\partial \eta} \frac{1}{2} \log \frac{R + (\xi + \eta)}{R - (\xi + \eta)} = \frac{1}{R} \frac{(\zeta^2 - \xi \eta)}{(\eta + \zeta^2)}$$

$$\frac{\partial}{\partial \eta} \frac{1}{b} \frac{1}{2} \log \frac{bR + (\xi + b^2 \eta)}{bR - (\xi + b^2 \eta)} = \frac{1}{R}$$

$$\frac{\partial}{\partial \eta} \tan^{-1} \frac{\zeta R}{\xi \eta - \zeta^2} = - \frac{1}{R} \frac{\zeta (\xi + \eta)}{(\eta + \zeta^2)}$$

$$R \frac{\partial}{\partial \zeta} R = (b^2 - 1) \zeta$$

$$\frac{\partial}{\partial \zeta} \frac{1}{2} \log \frac{R + (\xi + \eta)}{R - (\xi + \eta)} = - \frac{1}{R} \frac{\zeta (\xi + \eta)}{(\eta + \zeta^2)}$$

$$\frac{\partial}{\partial \zeta} \frac{1}{b} \frac{1}{2} \log \frac{bR + (\xi + b^2 \eta)}{bR - (\xi + b^2 \eta)} = - \frac{1}{R} \frac{\zeta (\xi + b^2 \eta)}{[\xi^2 + b^2 \zeta^2]}$$

$$\frac{\partial}{\partial \zeta} \tan^{-1} \frac{\zeta R}{\xi \eta - \zeta^2} = \frac{1}{R} \frac{\xi (\xi + b^2 \eta)}{[\xi^2 + b^2 \zeta^2]} + \frac{1}{R} \frac{\eta (\xi + \eta)}{(\eta + \zeta^2)}$$

APPENDIX B

SURFACE EDGE FORCES

LEADING EDGE POTENTIAL SUCTION

In the limit as the wing thickness goes to zero, the increasingly reduced pressure acting over a decreasing area results in a limiting suction force at the leading edge. If we consider the leading edge region, (figure 1) the force on the airfoil may be obtained by integrating over a control surface in the flow,

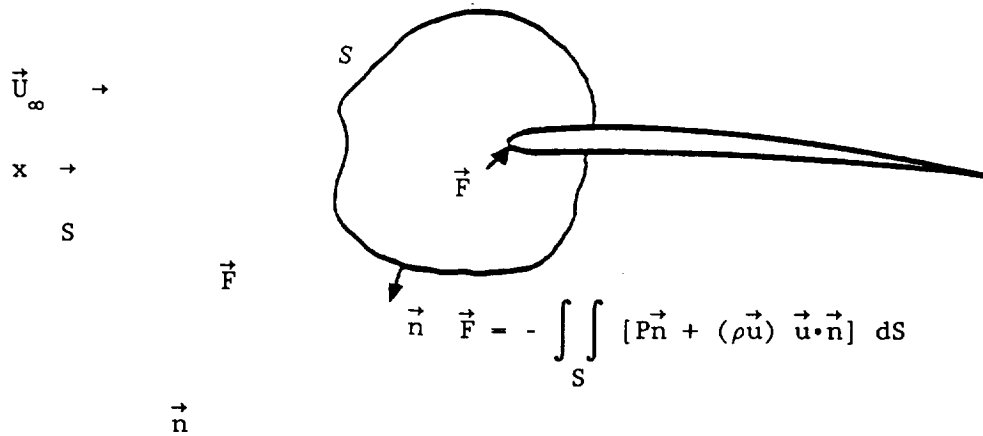


Figure 1. Leading Edge Suction Region.

where S is a control surface into which the leading edge penetrates and \vec{F} is the force on the area enclosed by S. In two dimensions the surface integral becomes a line integral and since for incompressible, irrotational flow

$$P = P_{\infty} - \frac{1}{2} \rho (u^2 + v^2)$$

$$\vec{n} dS = dy \vec{e}_x - dx \vec{e}_y$$

$$\begin{aligned} F_x &= - \int_C \left\{ \left[P_{\infty} - \frac{1}{2} \rho (u^2 + v^2) \right] dy + \rho u [u dy - v dx] \right\} \\ &= - \int_C P_{\infty} dy + \frac{1}{2} \rho \int_C [2uv dx + (v^2 - u^2) dy] \end{aligned}$$

where C is the contour around the leading edge of the airfoil and F_x is the force per unit of leading edge length.

As the wing thickness approaches zero, the wing becomes a line segment (figure 2) and the flow in the leading edge region is identical to the flow around a 180 degree corner. Incompressibly, it is described by

$$u_r = \frac{a}{\sqrt{r}} \cos(\theta/2) + U_\infty \cos \theta$$

$$u_\theta = -\frac{a}{\sqrt{r}} \sin(\theta/2) - U_\infty \sin \theta$$

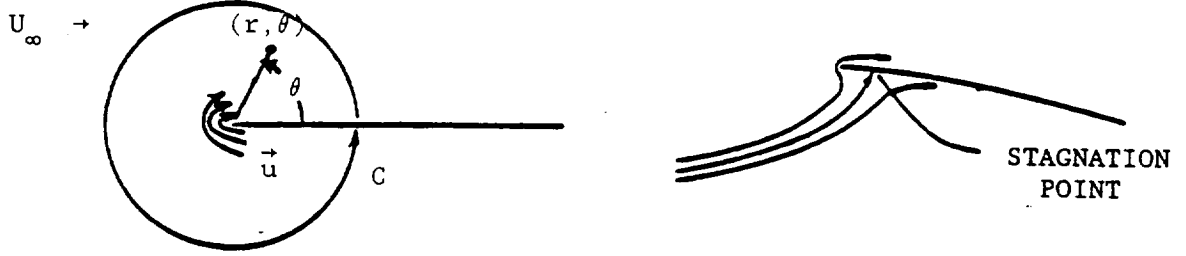


Figure 2. Wing Represented By Line Segment.

where (r, θ) is a coordinate system centered at the leading edge, and

$$u = u_r \cos \theta - u_\theta \sin \theta$$

$$v = u_r \sin \theta + u_\theta \cos \theta$$

$$dy = R \cos(\theta) d\theta$$

$$dx = -R \sin(\theta) d\theta$$

and C is the circle $r = R$.

Therefore, since

$$\int_C P_\infty dy = 0$$

as $R \rightarrow 0$

$$\sqrt{R} u = a \left[\cos\left(\frac{1}{2}\theta\right) \cos \theta + \sin\left(\frac{1}{2}\theta\right) \sin \theta \right] = a \cos\left(\frac{1}{2}\theta\right)$$

$$\sqrt{R} v = a \left[\cos\left(\frac{1}{2}\theta\right) \sin \theta + \sin\left(\frac{1}{2}\theta\right) \cos \theta \right] = a \sin\left(\frac{1}{2}\theta\right)$$

$$F_x = \rho a^2 \int_0^{2\pi} \left\{ -\cos\left(\frac{1}{2}\theta\right) \sin\left(\frac{1}{2}\theta\right) \sin \theta + \frac{1}{2} \left[\sin^2\left(\frac{1}{2}\theta\right) - \cos^2\left(\frac{1}{2}\theta\right) \right] \cos \theta \right\} d\theta$$

$$= -\frac{1}{2} \rho a^2 \int_0^{2\pi} \left[\sin^2 \theta + \cos^2 \theta \right] d\theta = -\pi \rho a^2 \quad (1)$$

To relate $-F_x$, the leading edge suction force, to the pressure distribution near the leading edge, the ΔC_p across the line segment must be evaluated.

$$\text{On the top} \quad \theta = 0 \quad u = \frac{a}{\sqrt{x}} + U_\infty, \quad v = 0$$

$$\text{On the bottom} \quad \theta = 2\pi \quad u = -\frac{a}{\sqrt{x}} + U_\infty, \quad v = 0$$

$$\Delta P = \frac{1}{2} \rho U_\infty \frac{4a}{\sqrt{x}}$$

$$\text{and if } c \text{ is the chord length} \quad \Delta C_p = \frac{4a}{U_\infty \sqrt{c}} \frac{1}{\sqrt{\xi}}; \quad \xi = x/c$$

$$C_t = \frac{\Delta \text{ THRUST}}{c \Delta z q_\infty} = \frac{-F_x}{c \frac{1}{2} \rho U_\infty^2} = \frac{2\pi}{c} \frac{a^2}{U_\infty^2} = \frac{\pi}{8} \left[\frac{4a}{\sqrt{c} U_\infty} \right]^2$$

$$\text{or} \quad \Delta C_p = \frac{A}{\sqrt{\xi}} \quad C_t = \frac{\pi}{8} A^2 \quad A = \frac{4a}{U_\infty \sqrt{c}}$$

These expressions relate the leading edge thrust coefficient to the net distribution, ΔC_p , at the leading edge.

In general we can write

$$C_p(\phi) = A_0 \cot\left(\frac{1}{2}\phi\right) + \sum_{n=1}^{\infty} A_n \sin(n\phi)$$

where

$$\xi = \frac{x}{c} = \frac{1}{2}(1 - \cos\phi) = \sin^2\left(\frac{1}{2}\phi\right)$$

$$\cot\left(\frac{1}{2}\phi\right) = \frac{\cos(\frac{1}{2}\phi)}{\sin(\frac{1}{2}\phi)} = \frac{\left[1 - \sin^2\left(\frac{1}{2}\phi\right)\right]^{1/2}}{\sin(\frac{1}{2}\phi)} = \left[\frac{1-\xi}{\xi}\right]^{1/2}$$

A_0 is the coefficient of the $\xi^{-1/2}$ term and therefore determines the leading edge suction force since only the term which is infinite at the leading edge contributes to the suction.

$$C_t = \frac{\pi}{8} A_0^2 \quad M_\infty = 0$$

For linearized compressible flow the following Mach number correction must be applied

$$C_t = \frac{\pi}{8} \beta A_0^2 \quad \beta^2 = 1 - M_\infty^2$$

To derive the expression for a swept wing, an infinitely skewed wing (figure 3) is considered

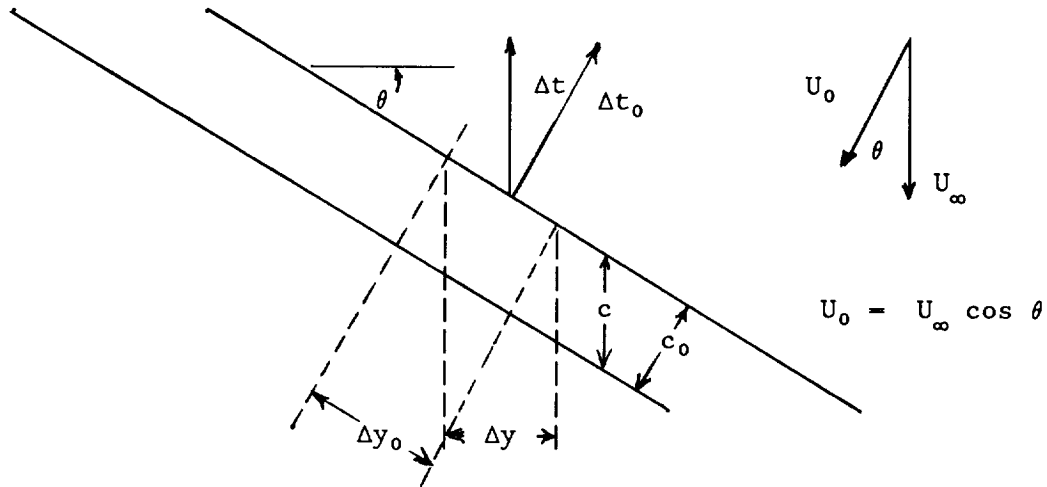


Figure 3. Infinitely Skewed Wing Representation.

Let the subscript or superscript ⁰ denote the variables normal to the leading edge. Then

$$\Delta C_p^0 = A_0 \cot(\phi/2) + \sum_{n=1}^{\infty} A_n \sin(n\phi)$$

and

$$C_{t_0} = \frac{\pi}{8} \beta_0 A_0^2 = \frac{\Delta t_0}{c_0 \Delta y_0 q_\infty^0}$$

the ratio of thrust per unit length is identical in either system

$$c_0 C_{t_0} q_\infty^0 = c C_t q_\infty = \frac{\Delta t_0}{\Delta y_0} = \frac{\Delta t}{\Delta y}$$

ΔC_p and C_t in the freestream coordinate system are based on freestream dynamic pressure q_∞ . Thus

$$q_\infty^0 = q_\infty \cos^2 \theta$$

$$\Delta C_p = \frac{\Delta C_p^0 q_\infty^0}{q_\infty} = \Delta C_p^0 \cos^2 \theta$$

and
$$C_{tc} = \frac{c_0 q_\infty^0}{q_\infty} C_{t_0} = C_{t_0} c_0 \cos^2 \theta$$

therefore
$$A_n = \dot{A}_n \cos^2 \theta$$

and
$$\begin{aligned} \beta_0^2 &= 1 - M_\infty^2 = \cos^2 \theta [1/\cos^2 \theta - M_\infty^2] = \cos^2 \theta [\tan^2 \theta + (1 - M_\infty^2)] \\ &= \cos^2 \theta [\tan^2 \theta + \beta^2] \end{aligned}$$

therefore combining terms

$$\begin{aligned} C_t(y) &= \frac{\Delta t}{c_0 \Delta y q_\infty} = \frac{\pi}{8} \beta_0 \dot{A}_0^2 \frac{q_\infty^0}{q_\infty} \frac{c_0}{c} \\ &= \frac{\pi}{8} \cos \theta [\tan^2 \theta + \beta^2]^{1/2} \frac{A_0^2 \cos^2 \theta \cos \theta}{\cos^4 \theta} \\ &= \frac{\pi}{8} [\tan^2 \theta + \beta^2]^{1/2} A_0^2 \end{aligned}$$

when ΔC_p is given by

$$\Delta C_p = A_0 \cot(\frac{1}{2}\phi) + \sum_{n=1}^{\infty} A_n \sin n\phi$$

SIDE EDGE POTENTIAL SUCTION

The method used to compute the suction force at surface tips is similar to that used for the leading edge. Since the flow is irrotational

$$\begin{aligned} \frac{\partial}{\partial y} \Delta u &= \frac{\partial}{\partial x} \Delta v \\ \frac{\partial}{\partial x} \Delta v(x,y) &= -\frac{1}{2} \frac{\partial}{\partial y} \Delta C_p(x,y) \end{aligned}$$

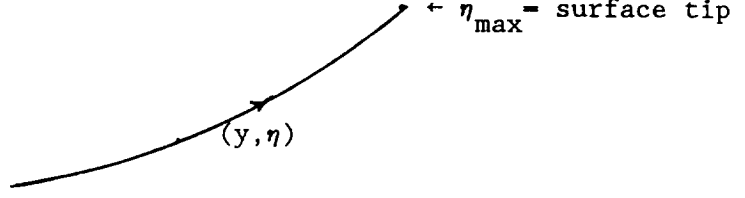
introduce a change of coordinates

let ξ be the fraction of chord

T be the slope of a constant ξ line $T = T(\xi, \eta)$

$$d\xi = \frac{1}{c} [dx - Tdy]$$

$$d\eta = dy$$



$$\left[\frac{\partial}{\partial x} \right]_y = \left[\frac{\partial \xi}{\partial x} \right]_y \left[\frac{\partial}{\partial \xi} \right]_\eta + \left[\frac{\partial \eta}{\partial x} \right]_y \left[\frac{\partial}{\partial \eta} \right]_\xi = \frac{1}{c} \left[\frac{\partial}{\partial \xi} \right]_\eta$$

$$\left[\frac{\partial}{\partial y} \right]_x = \left[\frac{\partial \xi}{\partial y} \right]_x \left[\frac{\partial}{\partial \xi} \right]_\eta + \left[\frac{\partial \eta}{\partial y} \right]_x \left[\frac{\partial}{\partial \eta} \right]_\xi = -\frac{T}{c} \left[\frac{\partial}{\partial \xi} \right]_\eta + \left[\frac{\partial}{\partial \eta} \right]_\xi$$

$$\begin{aligned} \text{or } \left[\frac{\partial}{\partial \xi} \right]_\eta \Delta v(\xi, \eta) &= \frac{1}{2} \left\{ T \left[\frac{\partial}{\partial \xi} \right]_\eta - c(\eta) \left[\frac{\partial}{\partial \eta} \right]_\xi \right\} \Delta C_p(\xi, \eta) \\ &= \frac{1}{2} \left[\frac{\partial}{\partial \xi} \right]_\eta \left[T(\xi, \eta) \Delta C_p(\xi, \eta) \right] \\ &\quad - \frac{1}{2} \left[\frac{\partial T}{\partial \xi} \right]_\eta \Delta C_p(\xi, \eta) - \frac{1}{2} c(\eta) \left[\frac{\partial}{\partial \eta} \right]_\xi \Delta C_p(\xi, \eta) \end{aligned}$$

then integrating

$$\Delta v(\xi, \eta) = \frac{1}{2} T(\xi, \eta) \Delta C_p(\xi, \eta) - \frac{1}{2} \int_0^\xi \left\{ \left[\frac{\partial T}{\partial \xi} \right]_\eta + c(\eta) \left[\frac{\partial}{\partial \eta} \right]_\xi \right\} \Delta C_p(\xi, \eta) d\xi \quad (2)$$

Near the tip, we assume a net pressure coefficient of the form

$$\Delta C_p(\xi, \eta) = \frac{1}{\eta_{\max}} \left[\frac{1}{2} (\eta_{\max}^2 - \eta^2) \right]^{1/2} \frac{c_{\text{avg}}}{c} C_{N_0} f(\xi), \quad \int_0^1 f(\xi) d\xi = 1$$

where

$$C_N \frac{c}{c_{\text{avg}}} = \frac{1}{\eta_{\max}} \left[\frac{1}{2} (\eta_{\max}^2 - \eta^2) \right]^{1/2} C_{N_0}$$

Differentiating

$$\left[\frac{\partial}{\partial \eta} \right]_\xi \Delta C_p(\xi, \eta) = -\frac{1}{2} \frac{\eta}{\eta_{\max}} \left[\frac{1}{2} (\eta_{\max}^2 - \eta^2) \right]^{-1/2} \frac{c_{\text{avg}}}{c} C_{N_0} f(\xi)$$

Then as $\eta \rightarrow \eta_{\max}$, equation 2 gives (keeping only the largest term)

$$\left[\eta_{\max} (\eta_{\max} - \eta) \right]^{-1/2}$$

$$\Delta v(\xi, \eta) = \frac{1}{2} \left[\eta_{\max} (\eta_{\max} - \eta) \right]^{-1/2} c_{\text{avg}} C_{N_0} \int_0^{\xi} f(\chi) d\chi$$

$$U_{\infty} \Delta v(\xi, \eta) = 2 a(\xi) \left[\eta_{\max} (\eta_{\max} - \eta) \right]^{-1/2}$$

$$\Rightarrow a(\xi) = \frac{1}{8} U_{\infty} \eta_{\max}^{-1/2} c_{\text{avg}} C_{N_0} \int_0^{\xi} f(\chi) d\chi$$

Using the expression derived for flow around a corner (equation 1) in conjunction with this relation, the suction force at the tip is given by

$$C_s(\xi) = \frac{\Delta F \eta}{c_T \Delta x q_{\infty}} = \frac{2\pi}{c_T q_{\infty}} \frac{1}{2} \rho_{\infty} a^2 = \frac{\pi}{32} \frac{c_{\text{avg}}}{c_T \eta_{\max}} C_{N_0}^2 \left[\int_0^{\xi} f(\chi) d\chi \right]^2$$

$$\frac{F \eta}{c_T^2 q_{\infty}} = \frac{\pi}{32} \frac{c_{\text{avg}}}{c_T \eta_{\max}} C_{N_0}^2 \int_0^1 \left\{ \int_0^{\xi} f(\chi) d\chi \right\}^2 d\xi$$

$$\frac{F \eta}{S_{\text{ref}} q_{\infty}} = \frac{\pi}{32} \frac{c_{\text{avg}} c_T}{S_{\text{ref}} \eta_{\max}} C_{N_0}^2 \int_0^1 \left\{ \int_0^{\xi} f(\chi) d\chi \right\}^2 d\xi$$

where c_T is the chord dimension at the tip.

EDGE FORCE AND MOMENT INCREMENTS

To account for edge vortex effects, the linearized forces and moments are corrected to reflect losses in suction and the associated formation of vortex forces for leading and side edges. The corrections are applied to the standard lift, side force and drag coefficients. The corresponding increments in the total moment coefficients are calculated by applying the above force increments at the appropriate x,y,z coordinates for the leading edge stations and center of pressure for the side edges.

For leading edge force calculations, the lost suction force for each span station is given by

$$C_s c \Delta s' (1 - K_s)$$

where C_s is the coefficient of leading edge suction, c is the local chord, $\Delta s'$ is the local span station width and K_s is the leading edge suction recovery factor. ($K_s = 1$ - full suction - no vortex) This force is subtracted from the direction normal to the section leading edge and re-entered as a force component rotated $\pm 90^\circ$ about the leading edge. The sign of the rotation is determined by the sign of the coefficient A_0 in the equation for leading edge suction.

The change in the total lift, side force and drag is calculated for each span station and is written as a function of four coordinate system rotations whose rotation angles are known from the leading edge geometry. The origin of each coordinate system is located at the leading edge of the section camber line.

The first transformation involves the rotation of the system (x_4, y_4, z_4) , whose x-axis is tangent to the local normal camber line, to the system (x_3, y_3, z_3) , whose x-axis is tangent to the corresponding chord plane as indicated in figure 4:

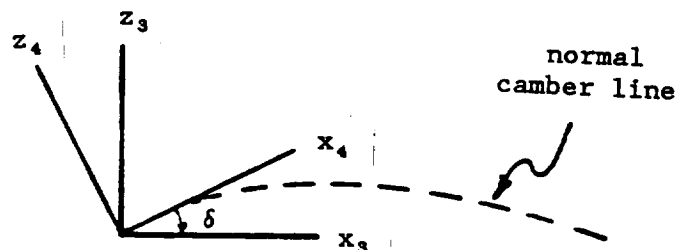


Figure 4. Axis of Rotation for First Transformation in Leading Edge Region.

where $\delta = -\tan^{-1} \left\{ [(dz/dx)_c + (dz/dx)_e + (dz/dx)_{\delta_F}] / \cos \Lambda \right\}$

$(dz/dx)_c$ is streamwise slope due to camber

$(dz/dx)_e$ is streamwise slope due to twist

$(dz/dx)_{\delta_F}$ is streamwise slope due to flap deflection

Λ is the local leading edge sweep angle.

The sweep term converts the total streamwise slope to a slope measured in the direction normal to the leading edge.

The two coordinates systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_{x_3} \\ C_{y_3} \\ C_{z_3} \end{Bmatrix} = \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix} \begin{Bmatrix} C_{x_4} \\ C_{y_4} \\ C_{z_4} \end{Bmatrix}$$

The second transformation involves the rotation of the system (x_3, y_3, z_3) , whose y-axis is tangent to the leading edge, to the system (x_2, y_2, z_2) , whose y-axis is normal to the configuration center line and in the plane of the surface (figure 5).

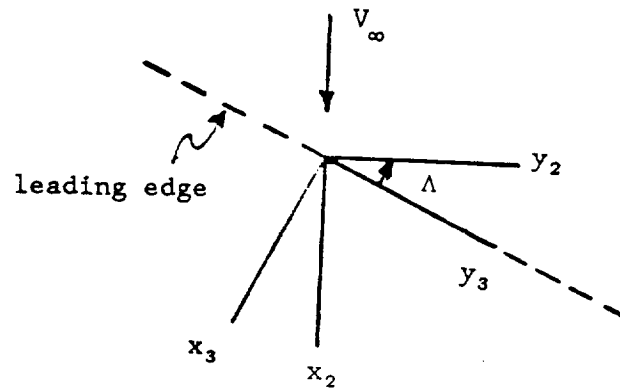


Figure 5. Axis of Rotation for Second Transformation in Leading Edge Region.

The two coordinates systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_{x_2} \\ C_{y_2} \\ C_{z_2} \end{Bmatrix} = \begin{bmatrix} \cos\Lambda & \sin\Lambda & 0 \\ -\sin\Lambda & \cos\Lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} C_{x_3} \\ C_{y_3} \\ C_{z_3} \end{Bmatrix}$$

The third transformation (figure 6) involves the rotation of the system (x_2, y_2, z_2) , whose z axis is normal to the local surface plane, to the system (x_1, y_1, z_1) , whose x , y_2 and z axes are in the body axes direction.

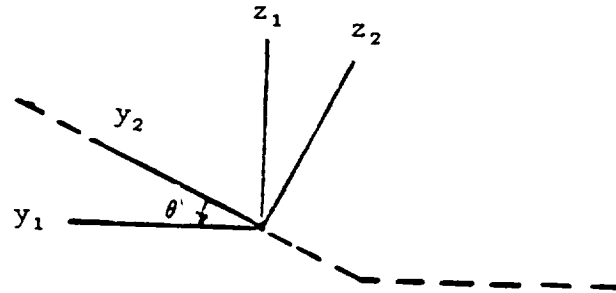


Figure 6. Axis of Rotation for Third Transformation in Leading Edge Region.

The rotation is about the x_2 , x_1 axis and of magnitude θ , the local dihedral angle. The two coordinate systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_{x_1} \\ C_{y_1} \\ C_{z_1} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} C_{x_2} \\ C_{y_2} \\ C_{z_2} \end{Bmatrix}$$

The fourth and final transformation (figure 7) involves the rotation of the body axis system (x_1, y_1, z_1) to the wind axes system (D, Y, L) .

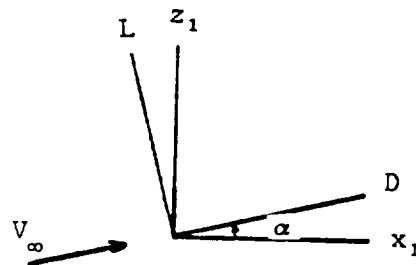


Figure 7. Axis of Rotation for Fourth Transformation in Leading Edge Region.

The rotation is about the (y_1, Y) -axis and of magnitude α , the angle of attack. The coordinate systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_D \\ C_Y \\ C_L \end{Bmatrix} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{Bmatrix} C_{x_1} \\ C_{y_1} \\ C_{z_1} \end{Bmatrix}$$

The composite transformation between the (x_4, y_4, z_4) coordinate system and (D, Y, L) coordinate system can then be expressed as

$$\begin{Bmatrix} C_D \\ C_Y \\ C_L \end{Bmatrix} = \begin{bmatrix} & & \\ & \Omega & \\ & & \end{bmatrix} \begin{Bmatrix} C_{x_4} \\ C_{y_4} \\ C_{z_4} \end{Bmatrix}$$

where Ω is the rotation matrix obtained from multiplication of the four previous specified transformation matrices.

Expressing C_{x_4} , C_{y_4} and C_{z_4} , in terms of the leading edge suction parameters,

$$C_{x_4} = C_s \cos \Delta s' (1 - K_s)$$

$$C_{y_4} = 0$$

$$C_{z_4} = A_0/|A_0| C_s \cos \Delta s' (1 - K_s)$$

we can now write the change in drag, side force and lift resulting from the force rotation at each span station:

$$\Delta C_D = C_s \cos \Delta s' (1 - K_s) \Omega_D$$

$$\Delta C_Y = C_s \cos \Delta s' (1 - K_s) \Omega_Y$$

$$\Delta C_L = C_s \cos \Delta s' (1 - K_s) \Omega_L$$

where

$$\Omega_D = [\cos \alpha (\cos \Lambda \cos \delta) + \sin \alpha (-\sin \theta \sin \Lambda \cos \delta + \cos \theta \sin \delta)]$$

$$+ A_0/|A_0| [-\cos \alpha (\cos \Lambda \sin \delta) + \sin \alpha (\sin \theta \sin \Lambda \sin \delta + \cos \theta \cos \delta)]$$

$$\Omega_Y = - [\cos \theta \sin \Lambda \cos \delta + \sin \theta \sin \delta]$$

$$+A_0/|A_0| [\cos \theta \sin \Lambda \sin \delta - \sin \theta \cos \delta]$$

and

$$\Omega_L = [-\sin \alpha (\cos \Lambda \cos \delta) + \cos \alpha (-\sin \theta \sin \Lambda \cos \delta + \cos \theta \sin \delta)]$$

$$+A_0/|A_0| [-\sin \alpha (-\cos \Lambda \sin \delta) + \cos \alpha (\sin \theta \sin \Lambda \sin \delta + \cos \theta \cos \delta)]$$

For side edge force calculations, the lost suction force at each chord station is given by

$$C_s c_T^2 \Delta(x/c)$$

where C_s is the coefficient of side edge suction, c_T is the tip chord and $\Delta(x/c)$ is the local nondimensional chord increment over which C_s is acting. This force is subtracted from the direction normal to the tip chord and re-entered as a force component rotated $\pm 90^\circ$ about the tip chord. The sign of the rotation is determined by the sign of the coefficient C_{N_0} in the equation for side edge suction.

In a manner similar to that for the leading edge forces, the change in the total lift, side force and drag coefficients is calculated for each chord increment and is written as a function of three coordinate system rotations whose angles are known from the tip geometry. The origin of each coordinate system is located on the chord line at the beginning of each chord increment.

The first transformation (figure 8) involves the rotation of the system (x_3, y_3, z_3) , whose X axis is parallel to the local camber line, to the system (x_2, y_2, z_2) , whose axis is tangent to the tip chord.

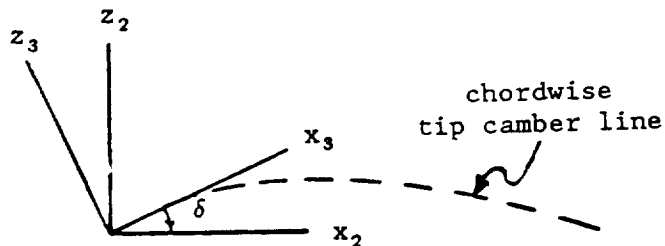


Figure 8. Axis of Rotation for First Transformation Along Chord.

where $\delta = \tan^{-1} [(dz/dx)_c + (dz/dx)_\epsilon + (dz/dx)_{\delta_F}]$
 $(dz/dx)_c$ is streamwise slope due to camber

$(dz/dx)_\epsilon$ is streamwise slope due to twist

and

$(dz/dx)_{\delta_F}$ is streamwise slope due to flap deflection

The two coordinate systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_{x_2} \\ C_{y_2} \\ C_{z_2} \end{Bmatrix} = \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix} \begin{Bmatrix} C_{x_3} \\ C_{y_3} \\ C_{z_3} \end{Bmatrix}$$

The second transformation (figure 9) involves the rotation of the system (x_2, y_2, z_2) , whose y-axis is normal to the tip chord, to the system (x_1, y_1, z_1) , whose x, y, and z-axes are in the body axes direction.

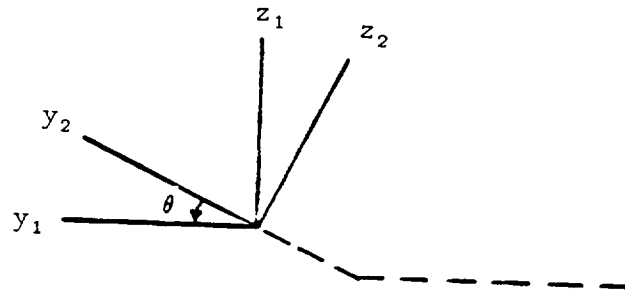


Figure 9. Axis of Rotation for Second Transformation Along Chord.

The rotation is about the (x_2, y_1) -axis and of magnitude θ , the local dihedral angle. The two coordinate systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_{x_1} \\ C_{y_1} \\ C_{z_1} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} C_{x_2} \\ C_{y_2} \\ C_{z_2} \end{Bmatrix}$$

The third and final transformation (figure 10) involves the rotation of the body axes system (x_1, y_1, z_1) to the wind axes system (D, Y, L) .

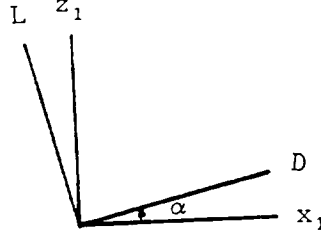


Figure 10. Axis of Rotation for Third Transformation Along Chord.

The rotation is about the (y_1, Y) -axis and of magnitude α , the angle of attack. The two coordinate systems are related by the following transformation matrix:

$$\begin{Bmatrix} C_D \\ C_Y \\ C_L \end{Bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{Bmatrix} C_{x_1} \\ C_{y_1} \\ C_{z_1} \end{Bmatrix}$$

The transformation between the (x_3, y_3, z_3) coordinate system and the (D, Y, L) coordinate system can then be expressed as

$$\begin{Bmatrix} C_D \\ C_Y \\ C_L \end{Bmatrix} = \begin{bmatrix} & & \\ & T & \\ & & \end{bmatrix} \begin{Bmatrix} C_{x_3} \\ C_{y_3} \\ C_{z_3} \end{Bmatrix}$$

where T is the rotation matrix obtained from multiplication of the three previously specified transformation matrices.

Expressing C_{x_3} , C_{y_3} and C_{z_3} in terms of the side edge suction parameters,

$$C_{x_3} = 0$$

$$C_{y_3} = -C_s c_T^2 \Delta(x/c)$$

$$C_{z_3} = (C_{N_0}/|C_{N_0}|) C_s c_T^2 \Delta(x/c)$$

we can now write the change in drag, side force and lift resulting from the force rotation at each side edge station:

$$\Delta C_D = C_s c_T^2 \Delta(x/c) T_D$$

$$\Delta C_Y = C_s c_T^2 \Delta(x/c) T_Y$$

$$\Delta C_L = C_s c_T^2 \Delta(x/c) T_L$$

where

$$T_D = \pm \sin(\alpha) \sin(\theta) + (C_{N_0}/|C_{N_0}|) [-\cos(\alpha) \sin(\delta) + \sin(\alpha) \cos(\theta) \cos(\delta)]$$

$$T_Y = \pm \cos(\theta) + (C_{N_0}/|C_{N_0}|) [\sin(\theta) \cos(\delta)]$$

$$T_L = \pm \cos(\alpha) \sin(\theta) + (C_{N_0}/|C_{N_0}|) [\sin(\alpha) \sin(\delta) + \cos(\alpha) \cos(\theta) \cos(\delta)]$$

The minus sign on the first term of each equation is for the right side of the configuration and the positive sign is for the left side. These force increments are numerically integrated along each tip chord to obtain the total change in lift, side force and drag due to side edge force rotation.

APPENDIX C

HYPERSONIC FINITE ELEMENT ANALYSIS

High Mach number analysis has a number of optional methods for calculating the pressure coefficient. In each method the only geometric parameter required is the element impact angle, δ , or the change in the angle of an element from a previous point.

The methods to be used in calculating the pressure in impact ($\delta > 0$) and shadow ($\delta < 0$) regions may be specified independently. A summary of the program pressure options is presented below.

<u>Impact Flow</u>	<u>Shadow Flow</u>
1. Modified Newtonian	1. Newtonian ($C_p = 0$)
2. Modified Newtonian+Prandtl-Meyer	2. Modified Newtonian+Prandtl-Meyer
3. Tangent wedge	3. Prandtl-Meyer from free-stream
4. Tangent-wedge empirical	4. OSU blunt body empirical
5. Tangent-cone empirical	5. Van Dyke Unified
6. OSU blunt body empirical	6. High Mach base pressure
7. Van Dyke Unified	7. Shock-expansion
8. Blunt-body skin friction model	8. Input pressure coefficient
9. Shock-expansion	9. Free molecular flow
10. Free molecular flow	
11. Input pressure coefficient	
12. Hankey flat-surface empirical	
13. Delta wing empirical	
14. Dahlem-Buck empirical	
15. Blast wave	
16. Modified tangent-cone	

C_L and C_D are in the stability axis system. Other coefficients are in the body reference coordinated system. It should also be noted that side force and pitching moment coefficients are invariant in an (α, β) transformation, whereas the yawing and rolling moment coefficients are not invariant.

A brief review of these methods will be presented in the following text.

MODIFIED NEWTONIAN

This method is probably the most widely used of all the hypersonic force analysis techniques. The major reason for this is its simplicity. Like all the force calculation methods, however, its validity in any particular application depends upon the flight condition and the shape of the vehicle

or component being considered. Its most general application is for blunt shapes at high hypersonic speed. The usual form of the modified Newtonian pressure coefficient is

$$C_p = K \sin^2 \delta$$

In true Newtonian flow ($M = \infty$, $\gamma = 1$) the parameter K is taken as 2. In the various forms of modified Newtonian theory, K is given values other than 2 depending on the type of modified Newtonian theory used. K is frequently taken as being equal to the stagnation pressure coefficient. In other forms it is determined by the following relationship (Reference 19).

$$K = C_{p_{\text{nose}}} / \sin^2 \delta_{\text{nose}}$$

where

$C_{p_{\text{nose}}}$ = the exact value of the pressure coefficient at the nose or leading edge

δ_{nose} = impact angle at the nose or leading edge

In other work K is determined purely on an empirical basis.

$$K = \text{fn } (M, \alpha, \text{shape})$$

When modified Newtonian theory is used, the pressure coefficient in shadow regions ($\delta < 0$) is usually set equal to zero.

MODIFIED NEWTONIAN PLUS PRANDTL-MEYER

This method, described as the blunt body Newtonian + Prandtl-Meyer technique, is based on the analysis presented by Kaufman in Reference 20. The flow model used in this method assumes a blunt body with a detached shock, followed by an expansion around the body to supersonic conditions. This method uses a combination of modified Newtonian and Prandtl-Meyer expansion theory. Modified Newtonian theory is used along the body until a point is reached where both the pressure and the pressure gradients match those that would be calculated by a continuing Prandtl-Meyer expansion.

The calculation procedure derived for determining the pressure coefficient using the blunt body Newtonian + Prandtl-Meyer technique is outlined below.

1. Calculate free-stream static to stagnation pressure ratio

$$P = P_{\infty}/P_0 = \left\{ 2/[(\gamma + 1) M_{\infty}^2] \right\}^{(\gamma/(\gamma-1))} \left\{ [2\gamma M_{\infty}^2 - (\gamma - 1)]/(\gamma + 1) \right\}^{(1/(\gamma-1))}$$

2. Assume a starting value of the matching Mach number, M_q (for $\gamma = 1.4$ assume $M_q = 1.35$)

3. Calculate matching point to free-stream static pressure ratio

$$Q = P_q/P_0 = \left\{ 2/[2 + (\gamma - 1)M_q^2] \right\}^{(\gamma/(\gamma-1))}$$

4. Calculate new free-stream static to stagnation pressure ratio

$$P_c = Q \left\{ 1 - (\gamma^2 M_q^4 Q)/[4(M_q^2 - 1)(1 - Q)] \right\}$$

5. Assume a new matching point Mach number (1.75) and repeat the above steps to obtain a second set of data.
6. With the above two tries use a linear interpolation equation to estimate a new matching point Mach number. This process is repeated until the solution converges.
7. Calculate the surface slope at the matching point

$$\sin^2(\delta_q) = (Q - P)/(1 - P)$$

8. Use the Prandtl-Meyer expansion equations to find the Mach number on the surface element, M_δ
9. Calculate the surface pressure ratio

$$P_\delta/P_0 = \eta_c [1 + (\gamma - 1)/(2) M_\delta^2]^{-(\gamma/(\gamma-1))}$$

where

η_c is provided as an empirical correction factor

P_δ is the pressure on the element of interest

10. Calculate the surface to free-stream pressure ratio

$$P_\delta/P_\infty = (1/P) (P_\delta/P_0)$$

11. Calculate the surface pressure coefficient

$$C_{P_\delta} = 2/(\gamma M_\infty^2) (P_\delta/P_\infty - 1)$$

The results of typical calculations using the above procedure are shown in Figure 1. Note that the calculations give a positive pressure coefficient at a zero impact angle. As pointed out in several references these results correlate well with test data for blunt shapes. However, if the surface curvature changes gradually to zero slope some distance from the blunt stagnation point the pressure calculated by this method will be too high. This is caused by characteristics near the nose intersecting the curved shock system and being reflected back onto the body. If the zero slope is reached near the nose (such as in a hemisphere or a cylinder) this effect has not had time to occur.

TANGENT-WEDGE

The tangent-wedge and tangent-cone theories are frequently used to calculate the pressures on two-dimensional bodies and bodies of revolution, respectively. These methods are really empirical in nature since they have no firm theoretical basis. They are suggested, however, by the results of more exact theories that show that the pressure on a surface in impact flow is primarily a function of the local impact angle. In this program the tangent-wedge pressures are calculated using the oblique shock relationships of NACA TR-1135 (Reference 21). The basic equation used is the cubic given by

$$[\sin^2(\theta_s)]^3 + b[\sin^2(\theta_s)]^2 + c[\sin^2(\theta_s)] + d = 0$$

or

$$R^3 + b R^2 + c R + d = 0$$

where

$$\theta_s = \text{shock angle}$$

$$\delta = \text{wedge angle}$$

$$b = -(M^2 + 2)/M^2 - \gamma \sin^2(\delta)$$

$$c = (2M^2 + 1)/M^4 + [(\gamma + 1)^2/4 + (\gamma - 1)/M^2] \sin^2(\delta)$$

$$d = -\cos^2(\delta)/M^4$$

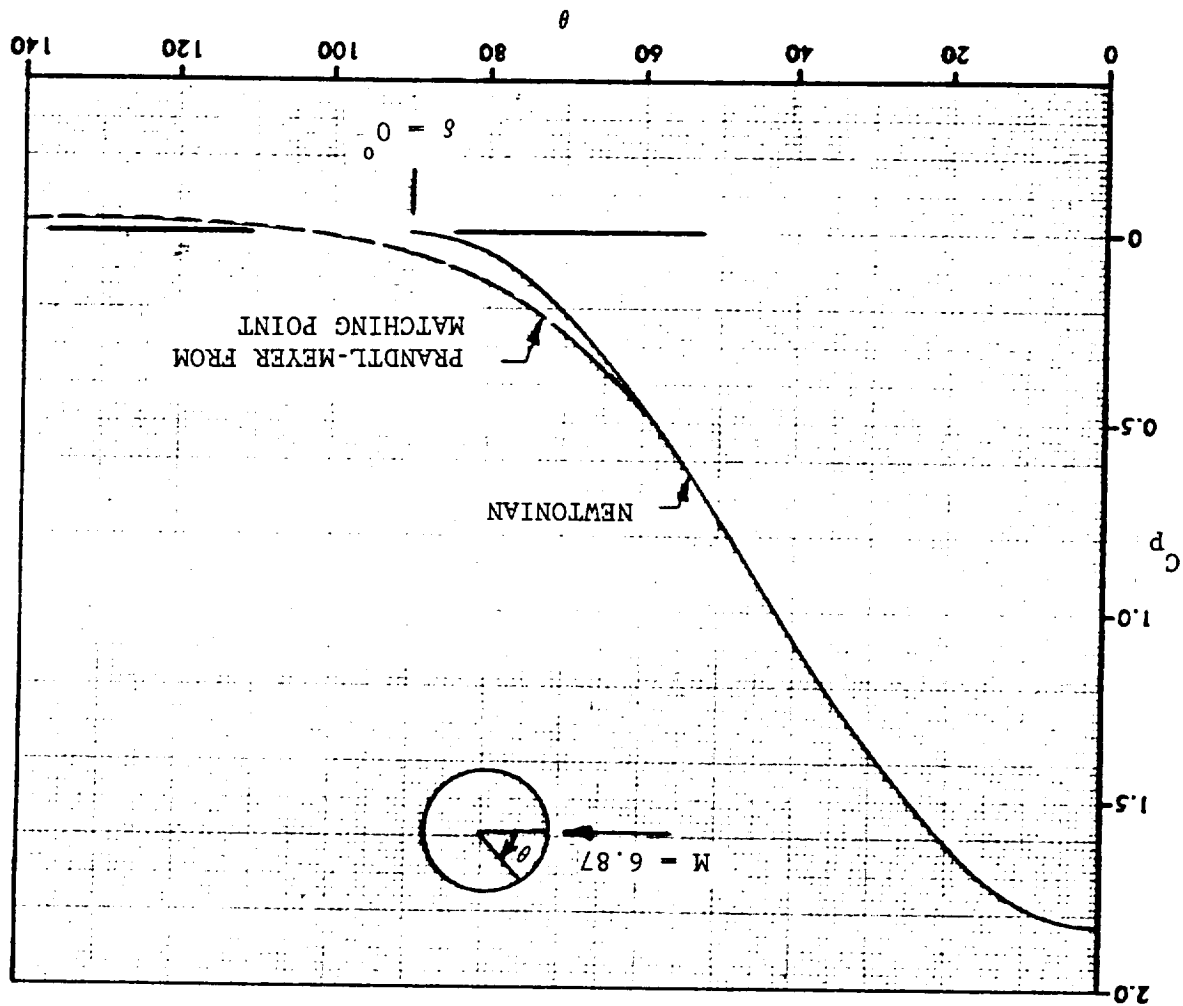


Figure 1. Blunt Body Newtonian + Prandtl-Meyer Pressure Results.

The roots of the above cubic equation may be obtained by using the trigonometric solution procedure (see Reference 22) as indicated below.

$$y_1 = 2 \sqrt{-p/3} \cos (w/3) - b/3$$

$$y_2 = -2 \sqrt{-p/3} \cos (w/3 + 60^\circ) - b/3$$

$$y_3 = -2 \sqrt{-p/3} \cos (w/3 - 60^\circ) - b/3$$

$$R_1 = y_1 - b/3$$

$$R_2 = y_2 - b/3$$

$$R_3 = y_3 - b/3$$

where

$$y_i = \text{roots of the reduced cubic equation}$$

$$p = -(b^2/3) + c$$

$$q = 2(b/3)^3 - bc/3 + d$$

$$\cos(w) = -q/(2\sqrt{-(p/3)^3})$$

$$R_i = \sin^2(\theta_s) = \text{roots of the cubic equation}$$

The smallest of the three roots corresponds to a decrease in entropy and is disregarded. The largest root is also disregarded since it never appears in physical actuality.

For small deflections, the cubic solution becomes very sensitive to numerical accuracy; that is, to the number of significant digits carried. Since this is dependent on the particular machine employed, an alternate procedure is used.

When the flow deflection angle is equal to or less than 2.0 degrees, the following equation is used instead of the above cubic relationships (Reference 23):

$$\sin^2(\theta_s) = 1/M^2 + (\gamma + 1)/(2) \delta / \sqrt{M^2 - 1}$$

Once the shock angle is obtained the remaining flow properties may be found from the relationship of Reference 21.

$$\begin{aligned} \text{density} &= \rho_2 = \rho \left\{ [6M^2 \sin^2(\theta_s)] / [M^2 \sin^2(\theta_s) + 5] \right\} \\ \text{temperature} &= T_2 = T \left\{ [7(M^2 \sin^2(\theta_s) - 1)(M^2 \sin^2(\theta_s) + 5)] / [36M^2 \sin^2(\theta_s)] \right\} \\ \text{pressure coefficient} &= C_p = \left\{ [7M^2 \sin^2(\theta_s) - 1] / 6 \right\} / (0.7M^2) \end{aligned}$$

where

$$(\quad)_2 = \text{conditions behind the shock}$$

Oblique shock detachment conditions are reached when no solution may be found to the above cubic relationships. Under these conditions the program uses the Newtonian + Prandtl-Meyer method for continued calculations.

TANGENT-WEDGE, TANGENT-CONE, AND DELTA WING NEWTONIAN EMPIRICAL METHOD

The tangent-cone and the tangent-wedge (figure 2) Newtonian empirical methods used in this program are based on the empirical relationships derived below.

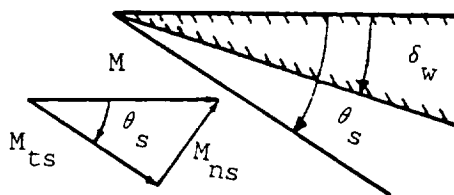


Figure 2. Tangent-Cone and Wedge Notations.

For wedge flow

$$\sin(\theta_s) = \sin(\delta_w) / [(1 - \epsilon) \cos(\theta_s - \delta_w)]$$

where

$$\epsilon = \rho / \rho_2 = (\gamma - 1) / (\gamma + 1) \left\{ 1 + 2 / [(\gamma - 1) M_{ns}^2] \right\}$$

For cone flow (thin shock layer assumption)

$$\sin(\theta_s) \approx \sin(\delta_c) / [(1 - \epsilon/2)\cos(\theta_s - \delta_c)]$$

In the limit as $M \rightarrow \infty$, $\epsilon = \epsilon_{\lim} = (\gamma - 1)/(\gamma + 1)$ and $\cos(\theta_s - \delta) = 1$

Therefore

<u>wedge</u>	<u>cone</u>
$\sin(\theta_s) = (\gamma + 1)/2 \sin(\delta_w)$	$\sin(\theta_s) = 2(\gamma + 1)/(\gamma + 3) \sin(\delta_c)$

These limiting expressions for θ may now be compared with the data of TR-1135 (Reference 21) at $\gamma = 7/5$ using the following similarity parameters. The exact equations contain three variables - θ_s , δ , and ϵ . Noting that for $\gamma = \text{constant}$, $\epsilon = \text{fn}(M_{\text{ns}})$ only, the preceding equations may be rewritten in the following form:

<u>wedge</u>	<u>cone</u>
$M_{\text{ns}} = M \sin(\delta_w) / [(1 - \epsilon)\cos(\theta_s - \delta_w)]$	$M_{\text{ns}} = M \sin(\delta_w) / [(1 - \epsilon)\cos(\theta_s - \delta_w)]$

The parameter $(\theta - \delta)$ is approximately constant and independent of M except near the shock detachment condition. The equations essentially contain only two variables, M_{ns} and $M \sin \delta$. These are used as coordinates to plot the data for wedge flow shown in Figure 3. A similar plot could be obtained for cone flow. From the figure it is seen that the data are nearly normalized with the use of these coordinates.

For rapid calculation we need relationships for M_{ns} as a function of $M \sin(\delta)$ that satisfy the following requirements:

1. The effect of shock detachment is neglected
2. At $M \sin(\delta) = 0$, $M_{\text{ns}} = 1$
3. The solution asymptotically approaches the $M = \infty$ line
4. Have the correct slope, $d[M_{\text{ns}}] / d[M \sin(\delta)]$ at $M \sin(\delta) = 0$

These conditions lead to equations of the following form

$$\text{wedge } M_{\text{ns}} = K_w M' + e^{-(K_w M'/2)}$$

$$K_w = (\gamma + 1)/2$$

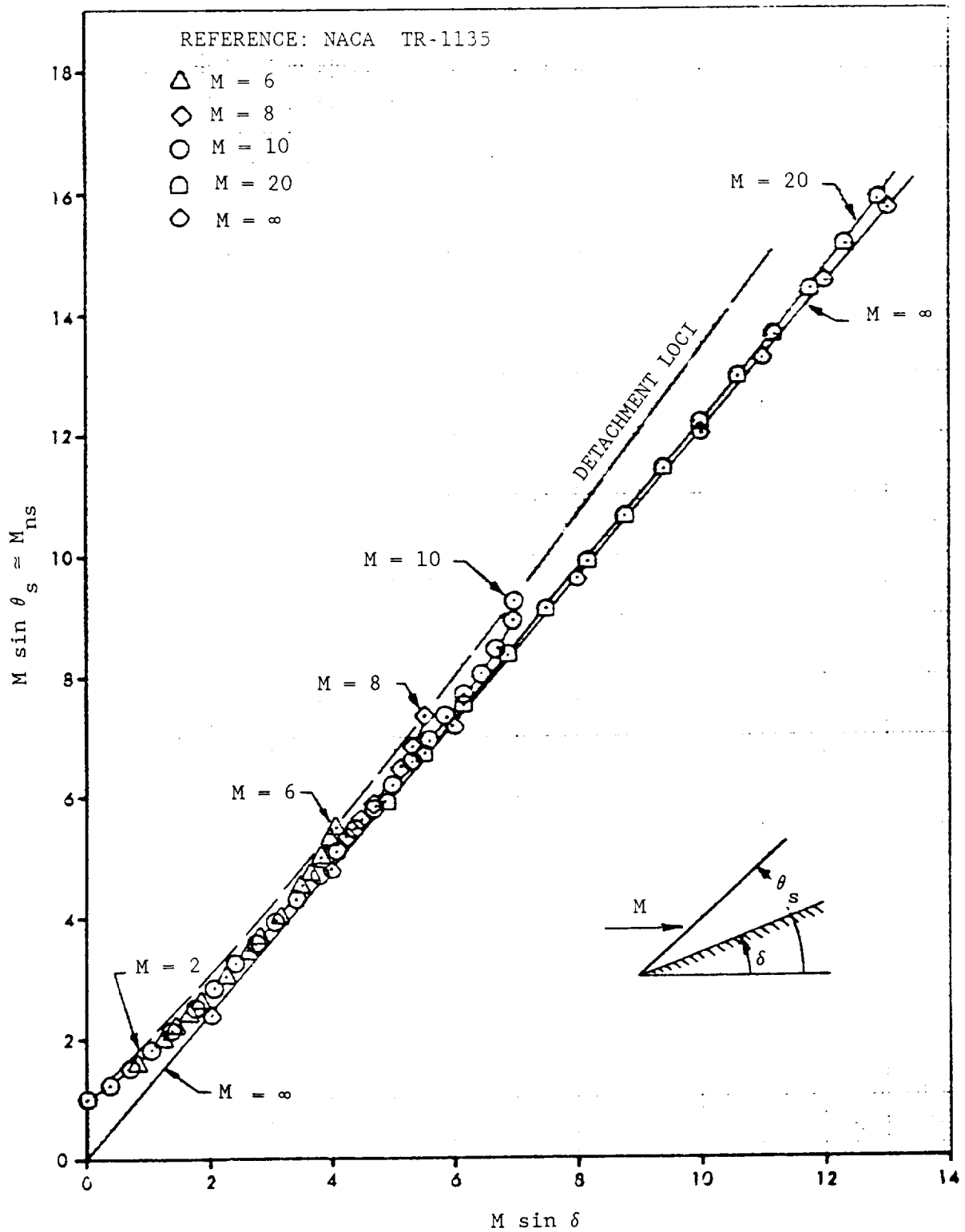


Figure 3. Wedge Flow Shock Angle.

$$\text{cone } M_{ns} = K_w M' + e^{-(K_c M')}$$

where

$$M' = M \sin(\delta)$$

$$K_c = 2(\gamma + 1)/(\gamma + 3)$$

These expressions are compared with the data of TR-1135 in Figures 4 and 5. The cone data are also shown in Figure 6 with the same scales as in Figure 3.

The pressure coefficient may now be obtained by the following relationships for a wedge and cone respectively.

$$C_p = 4/(\gamma + 1)(M_{ns}^2 - 1)/M^2$$

$$C_p = 2\sin^2(\delta) \left\{ 1 - [(\gamma - 1)M_{ns}^2 + 2]/[4(\gamma + 1)M_{ns}^2] \right\}^{-1}$$

Experimental results have shown the pressure on the centerline of a delta wing to be in agreement with two-dimensional theory at small values of the similarity parameter ($M' < 3.0$) and with conical flow theory at higher values. The previous expressions derived for wedge and cone flows have been combined to give these features. The resulting relationships are given below.

$$M_{ns} = K_c M' + e^{-(K_c - K_w/2)M'}$$

For $\gamma = 7/5$

$$M_{ns} = 1.09M\sin(\delta) + e^{-(0.49M\sin(\delta))}$$

The similarity parameter relationship for pressure is

$$M^2 C_p = (4/(\gamma + 1))(M_{ns}^2 - 1)$$

The shock angle and pressure coefficient calculated from the above equations are compared with the experimental results (Reference 28) in Figures 7 and 8 respectively.

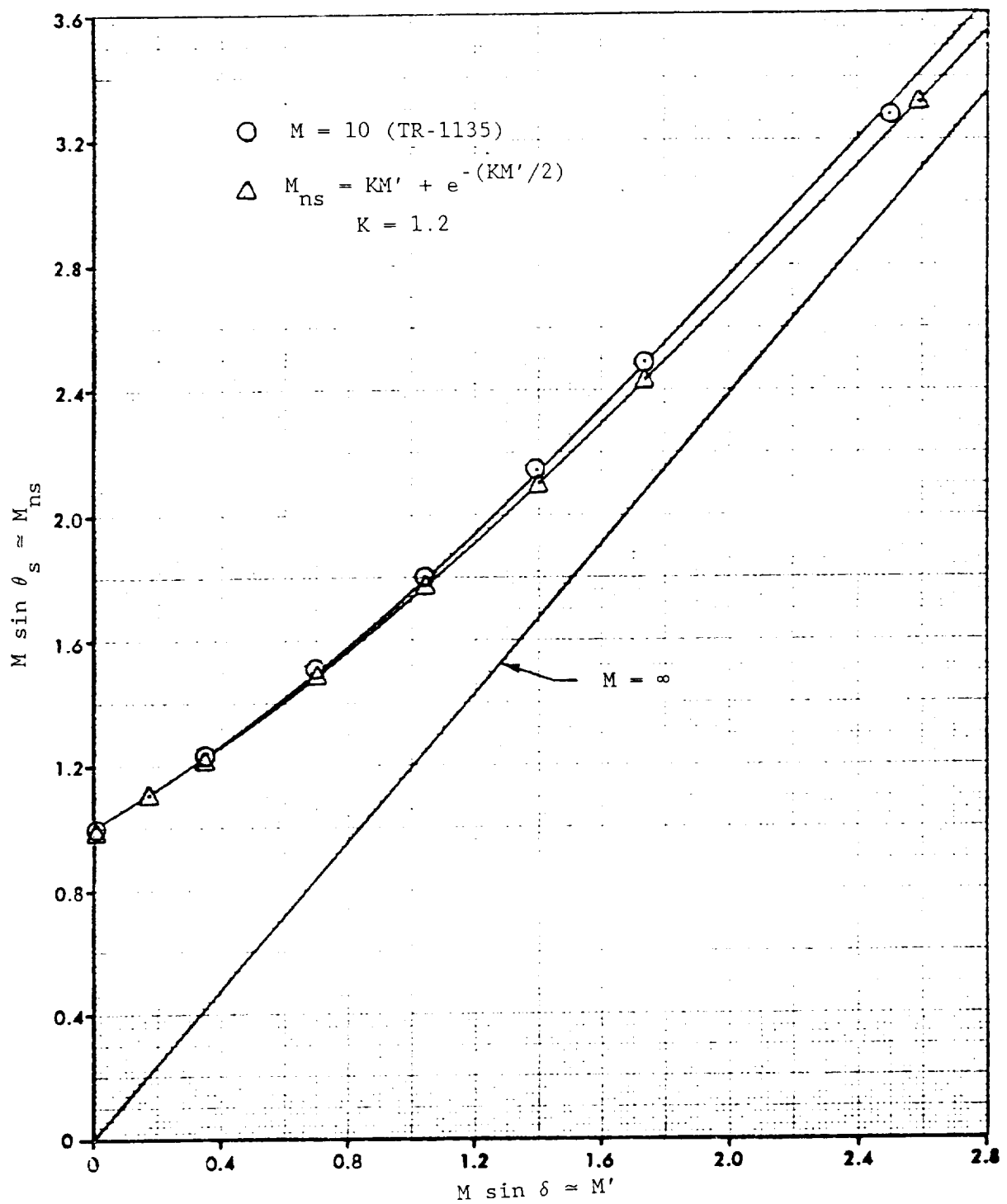


Figure 4. Wedge Flow Shock Angle Empirical Correlation.

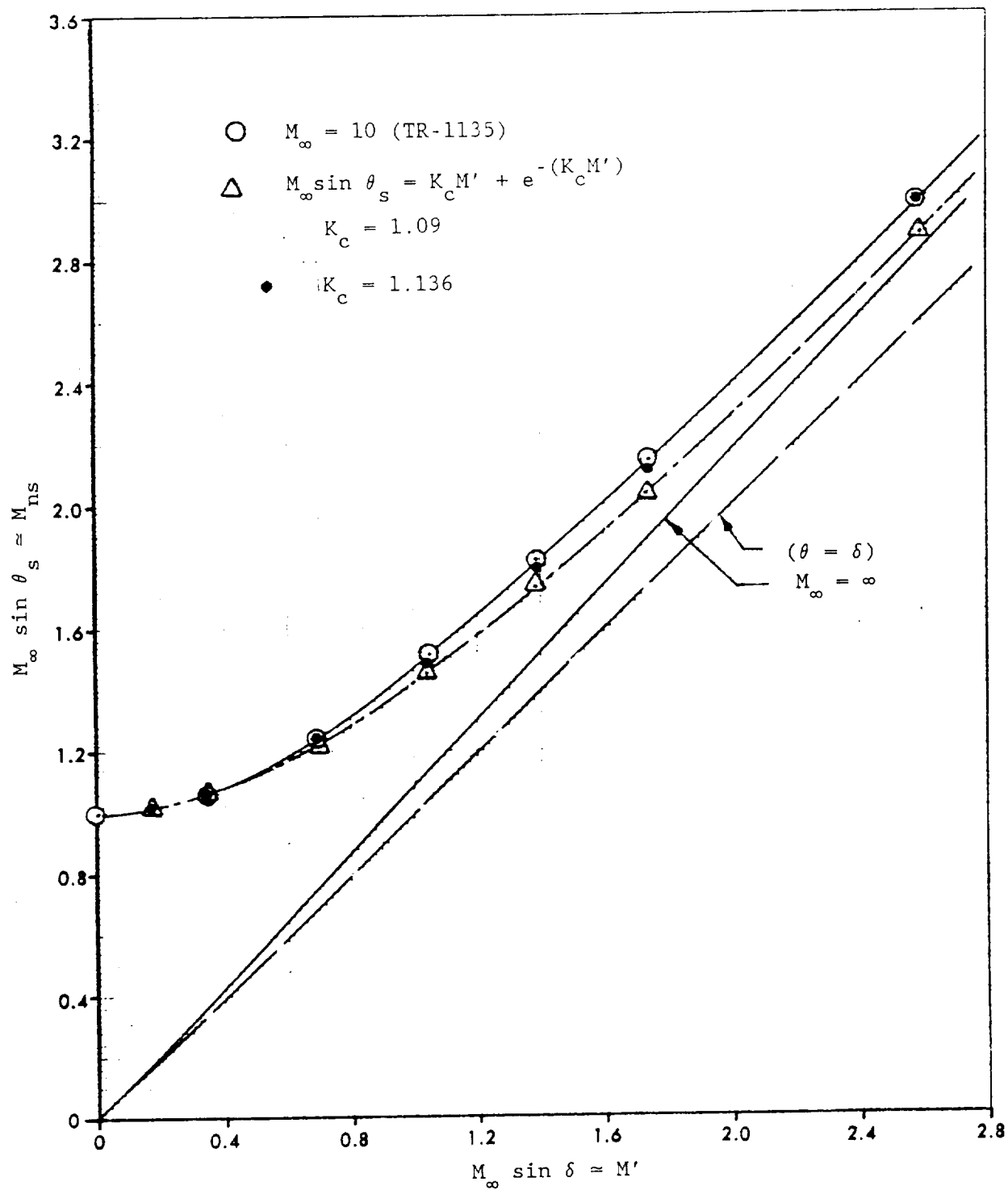


Figure 5. Conical Flow Shock Angle Empirical Correlation.

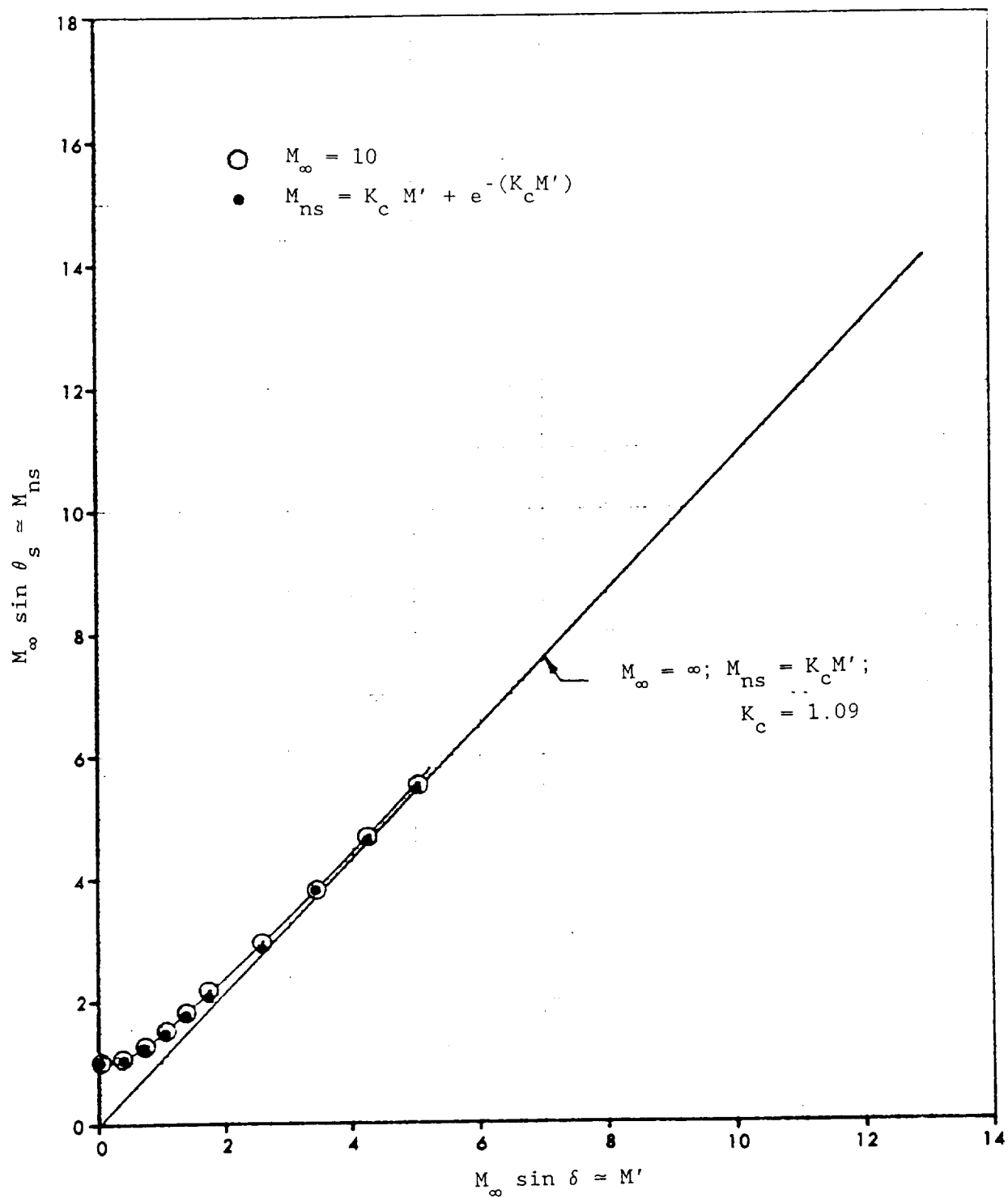


Figure 6. Conical Flow Shock Angle Empirical Correlation.

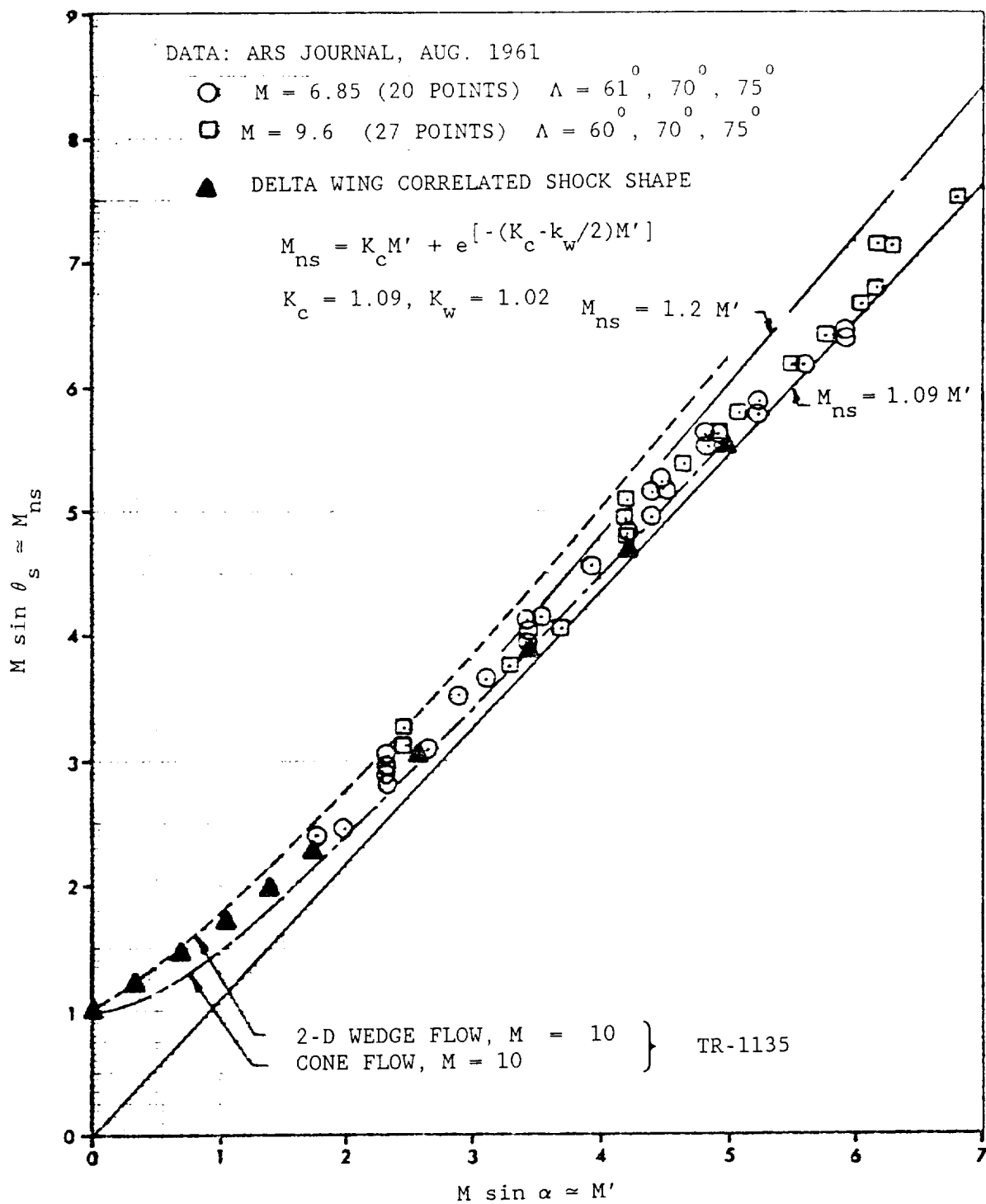


Figure 7. Delta Wing Centerline Shock Angle Correlation.

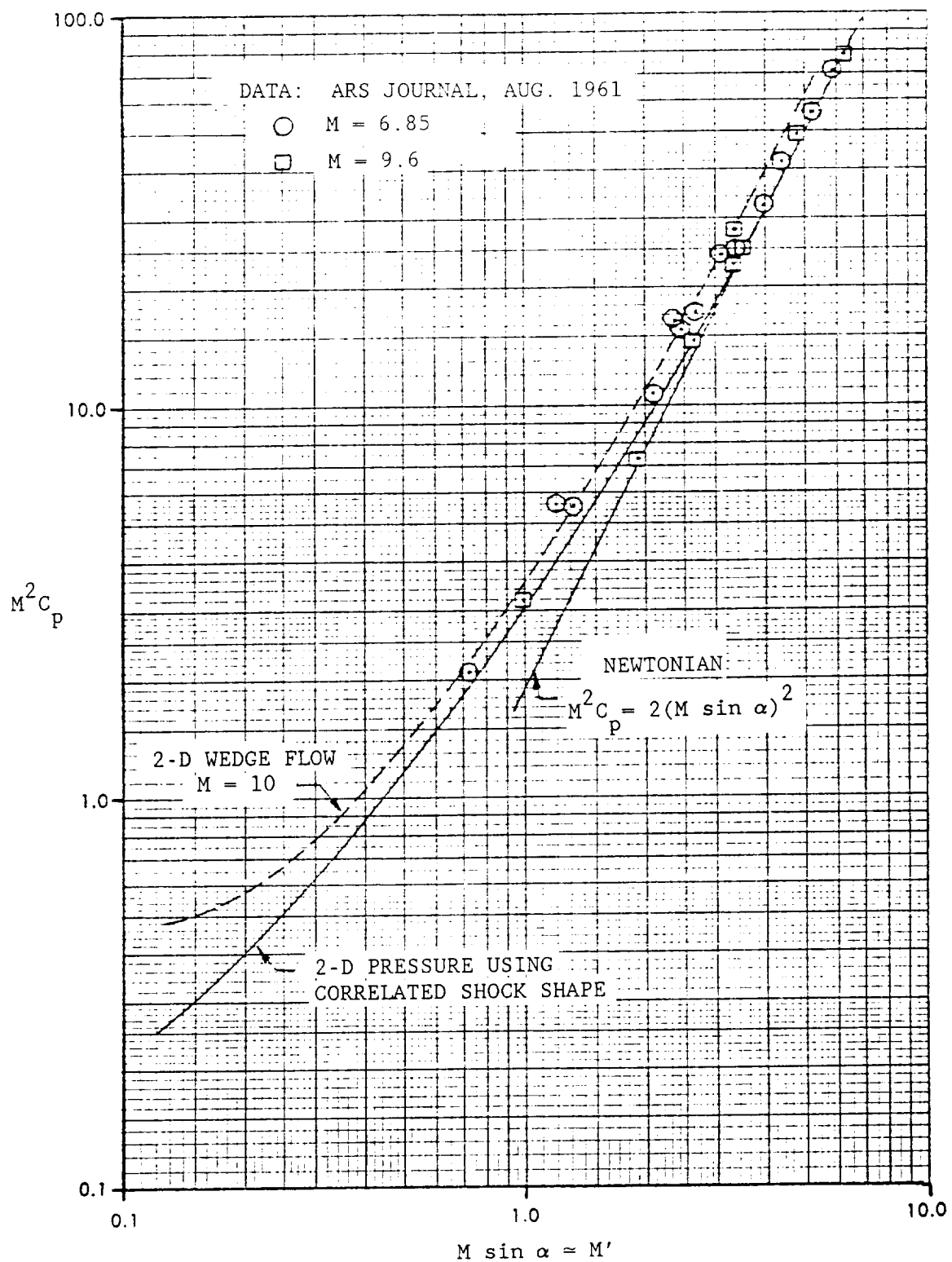


Figure 8. Delta Wing Centerline Pressure Coefficient Correlation.

OSU BLUNT BODY EMPIRICAL METHOD

The OSU (Ohio State University) blunt body empirical equation describes the pressure distribution about cylinders in supersonic flow. The equation was presented in Reference 25 and was stated to match "all the data obtained on the cylinders in the present test series with a maximum deviation of 2.5 percent." The expression used is

$$P_1/P_{t_\infty} = 0.32 + 0.455 \cos(\theta) + 0.195 \cos(2\theta) + 0.035 \cos(3\theta) - 0.005 \cos(4\theta)$$

where

θ = peripheral angle on a cylinder
(= 0 at the stagnation point) = $(90^\circ - \delta)$

P_1 = surface pressure

P_{t_∞} = total pressure rise through normal shock

The pressure coefficient is calculated from the relationship

$$C_p = [(P_1/P_{t_\infty})(P_{t_\infty}/P_\infty) - 1]/(\gamma M^2/2)$$

where

$$P_{t_\infty}/P_\infty = K\gamma M^2/2 + 1$$

K = stagnation pressure coefficient = $C_{p_{stag}}$

P_∞ = freestream pressure

γ = ratio of specific heats = 1.4

VAN DYKE UNIFIED METHOD

This force calculation method is based on the unified supersonic-hypersonic small disturbance theory proposed by Van Dyke in Reference 26 as applied to basic hypersonic similarity results. The method is useful for thin profile shapes and as the name implies extends down to the supersonic speed region.

The similarity equations that form the basis of this method are derived by manipulating the oblique shock relations for hypersonic flow. The basic derivations are shown on pages 753 and 754 of Reference 31. The result

obtained for a compression surface under the assumption of a small deflection angle and large Mach number is (hypersonic similarity equation).

$$C_p = \delta^2 [(\gamma + 1)/2 + \sqrt{((\gamma + 1)/2)^2 + 4/H^2}]$$

where H is the hypersonic similarity parameter given by $M\delta$. The contribution by Van Dyke in Reference 26 suggests that this relationship will also be valid in the realm of supersonic linear theory if the

hypersonic similarity parameter $(\sqrt{M^2 - 1})\delta$. This latter parameter is used in the calculations for this force option in the arbitrary body program.

A similar method may also be obtained for a surface in expansion flow with no leading edge shock such as on the upper side of an airfoil. The resulting equation is

$$C_p = \delta^2 [2/(\gamma H^2)] \left\{ [(1 - (\gamma - 1)H/2)]^{(2\gamma/(\gamma-1) - 1)} \right\}$$

where again H is taken to be $(\sqrt{M^2 - 1})\delta$ in the unified theory approach.

SHOCK-EXPANSION METHOD

This force calculation method is based on classical shock-expansion theory (see Reference 27). In this method the surface elements are handled in a "strip-theory" manner. The characteristics of the first element of each longitudinal strip of elements may be calculated by oblique shock theory, by conical flow theory, or by a Prandtl-Meyer expansion. Downstream of this initial element the forces are calculated by a Prandtl-Meyer expansion.

By a proper selection of the element orientation the method may be used for both wing-like shapes and for more complex body shapes. In this latter case the method operates in a hypersonic shock-expansion theory mode.

FREE MOLECULAR FLOW METHOD

At very high altitudes conventional continuum flow theories fail and one must begin to consider the general macroscopic mass, force, and energy transfer problem at the body surface. This condition occurs when the air is sufficiently rarefied so that the mean free path of the molecules is much greater than a characteristic body dimension. This condition is known as free molecular flow and the method of analysis selected for this program is described in Reference 28. This method was also used in Reference 29. The equations used were taken from these references and are presented below.

Pressure coefficient

$$C_p = S^{-2} \left\{ \left[(2-f_n)/\sqrt{\pi} S \sin(\delta) + f_n/2 \sqrt{T_b/T_\infty} \right] e^{-(S \sin(\delta))^2} + \left[(2-f_n)(S^2 \sin^2(\delta) + 1/2) + f_n/2\sqrt{\pi} \sqrt{T_b/T_\infty} S \sin(\delta) \right] [1 + \operatorname{erf}(S \sin(\delta))] \right\}$$

Shear force coefficient

$$C_f = [\cos(\delta) f_t] / (\sqrt{\pi} S) \left\{ e^{-(S \sin(\delta))^2} + \sqrt{\pi} S \sin(\delta) [1 + \operatorname{erf}(S \sin(\delta))] \right\}$$

where

$$S = \text{speed ratio} = \sqrt{\gamma/2} M_\infty$$

$$f_n = \text{normal momentum accommodation coefficient}$$

1.0 for Newtonian
0.0 for completely diffuse reflection)

$$\delta = \text{impact angle}$$

$$T_b = \text{body temperature, } ^\circ \text{K}$$

$$T_\infty = \text{free-stream temperature, } ^\circ \text{K}$$

$$\operatorname{erf} = \text{error function } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

$$f_t = \text{tangential momentum accommodation coefficient}$$

0.0 for Newtonian flow
1.0 for completely diffuse reflection

The pressure force acts perpendicular to the surface and this direction is readily obtained since the element normal has already been determined in the geometry subroutines. The shear force acts in the direction of the tangential velocity component on the surface and this direction is determined by taking successive vector products. The procedure is illustrated in figure 9 where the incident velocity vector is defined as

$$\vec{V} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}$$

and the surface normal as

$$\vec{N} = N_X \vec{i} + N_Y \vec{j} + N_Z \vec{k}$$

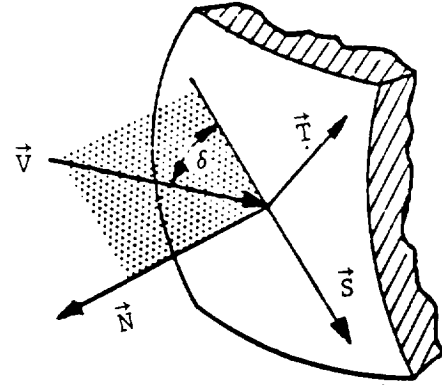


Figure 9. Force Components on a Surface

First, a surface tangent vector (\vec{T}) is defined by the cross product of the normal and velocity vectors;

$$\vec{T} = T_X \vec{i} + T_Y \vec{j} + T_Z \vec{k}$$

where

$$T_X = N_Y V_Z - N_Z V_Y$$

$$T_Y = N_Z V_X - N_X V_Z$$

$$T_Z = N_X V_Y - N_Y V_X$$

Then the direction of the shear force (\vec{S}) is given by the cross product of the surface tangent and normal vectors;

$$\vec{S} = S_X \vec{i} + S_Y \vec{j} + S_Z \vec{k}$$

where

$$S_X = T_Y N_Z - T_Z N_Y$$

$$S_Y = T_Z N_X - T_X N_Z$$

$$S_Z = T_X N_Y - T_Y N_X$$

The final components of the shear force in the vehicle axis system are given by

$$\text{SHEAR}_X = (\text{SHEAR}) (S_X) / \text{STOTAL}$$

$$\text{SHEAR}_Y = (\text{SHEAR}) (S_Y) / \text{STOTAL}$$

$$\text{SHEAR}_Z = (\text{SHEAR}) (S_Z) / \text{STOTAL}$$

where

SHEAR is the shear force as calculated by the free molecular flow equations.

$$\text{STOTAL} = (S_X^2 + S_Y^2 + S_Z^2)^{1/2}$$

In using the free molecular flow method the above analysis must be carried out over the entire surface of the shape including the base, shadow regions, etc. When the free molecular flow method is selected, it is used for both impact and shadow region.

The plane formed by the velocity vector and the surface normal is referred to as the velocity plane (shaded region in the sketch), since both the incident and surface velocity are in this plane. This definition is correct for two-dimensional flow, however, it is only an approximation to the shear direction in the general arbitrary-body case.

HANKEY FLAT-SURFACE EMPIRICAL METHOD

This method uses an empirical correlation for lower surface pressures on blunted flat plates. The method, derived in Reference 30, approximates tangent-wedge at low impact angles and approaches Newtonian at high impact angles. The pressure coefficient is given by

$$C_p = 1.95 \sin^2(\delta) + 0.21 \cos(\delta) \sin(\delta)$$

DAHLEM-BUCK EMPIRICAL METHOD

This is an impact method that has been derived such that tangent-cone and Newtonian results are approximated, respectively, at low and high values of the impact angle. The empirical relationships presented in Reference 31 are

$$\text{for } \delta < 22.5^\circ \quad C_p = \left\{ 1 + [\sin(4\delta)]^{3/4} \right\} \sin(\delta)^{5/4} / [4\cos(\delta)\cos(2\delta)]^{3/4}$$

$$\text{for } \delta \geq 22.5^\circ \quad C_p = 2.0 \sin^2(\delta)$$

BLAST WAVE PRESSURE INCREMENTS

This method uses conventional blast-wave parameters to calculate the overpressure due to bluntness effects. Force contributions determined by this procedure must be added to the regular inviscid pressure forces (tangent-wedge, tangent-cone, Newtonian, etc.) calculated over the same vehicle geometry. The specific blast wave solutions used in the Program were derived by Lukasiewicz in Reference 32:

$$P/P_{\infty} = A M_{\infty}^2 \left\{ (C_D)^{(1/(1+j))} / [(X_0 - X)/d] \right\}^{(2+j)/3} + B$$

where

C_D is the nose drag coefficient

d is the nose diameter or thickness

X_0 is a coordinate reference point

and the coefficients A, B are

Flow	j	A	B
Two dimensional	0	0.121	0.56
Axisymmetric	1	0.067	0.44

MODIFIED TANGENT-CONE METHOD

This method, originally developed for use on cones with elliptical cross sections, modifies the tangent-cone result by an increment representing the deviation from an average pressure divided by an average Mach number. More specifically, the following equations are used (after Jacobs, Reference 33):

$$C_p = C_{p_{tc}} - (C_{p_{tc}} - C_{p_{avg}}) / M_{avg}$$

where C_p is the surface pressure coefficient

$C_{p_{tc}}$ is the conventional tangent-cone pressure coefficient

$C_{p_{avg}}$ is the average pressure coefficient

$$= \frac{\sum C_{p_t} A}{\sum A}, \quad A \text{ is element area}$$

M_{avg} is the average Mach number, defined for an equivalent cone having pressure coefficient $C_{p_{avg}}$.

HIGH MACH BASE PRESSURES

For a body in high speed flow it might be expected that any base regions would experience total vacuum. That is,

$$C_p = -1/(\gamma M_\infty^2/2)$$

However, the viscosity of real gases causes some pressure to be felt in base region and experimental data have shown this to be roughly 70% vacuum for air. Therefore, the expression

$$C_p = -1/M_\infty^2$$

has been included in the program.

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16. Abstract An aerodynamic analysis system based on linear potential theory at subsonic / supersonic speed and newtonian impact type finite element solutions at hypersonic conditions is described. Three dimensional configurations having multiple non-planar surfaces of arbitrary planform and bodies of non-circular contour may be analyzed. Static, rotary and control longitudinal and lateral directional characteristics may be generated. The analysis has been implemented on a time sharing system in conjunction with an input tablet digitizer and an interactive graphics input/output display and editing terminal to maximize its responsiveness to the preliminary analysis problem. CDC 175 computational time of 45 CPU seconds/ Mach number at subsonic - supersonic speeds and 1 cpu second/ Mach number / altitude at hypersonic conditions for a typical simulation indicates that the program provides an efficient analysis tool for systematically performing various aerodynamic configuration tradeoff and evaluation studies.					
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